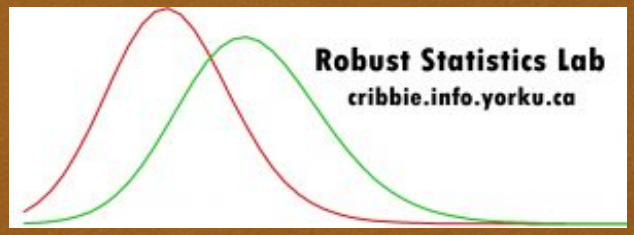
Equivalence Testing: A *Soon-To-Be* Household Name in Psychology?

Rob Cribbie



Collaborators

- I have been blessed with fantastic undergraduate and graduate students who deserve much of the credit for the work I will be presenting:
  - Teresa Allan (PhD Student, Psychology, University of Ottawa)
  - Nataly Beribisky (MA Student, QM Program, York University)
  - Alyssa Counsell (Assistant Professor, Psychology, Ryerson University)
  - Heather Davidson (Biostatistician, Sunnybrook)
  - Linda Farmus (MA Student, QM Program, York University)
  - Jason Goertzen (Senior Consultant - Research, Alberta Health Service)
  - Naomi Gutierrez (Undergraduate Student, Psychology, York)
  - Joseph Hoyda (Undergraduate Student, Psychology, York)
  - Jamie Kim (Independent Researcher)
  - Constance Mara (Assistant Professor, Division of Behavioral Medicine and Clinical Psychology, Cincinnatti Children's Hospital)
  - Victoria Ng (Data Analyst, Mental Health Systems)

Equivalence Tests

- Many empirical questions in psychological research involve a lack of relationship among variables
  - For example, a researcher may be interested in demonstrating that a clinical group subjected to a therapeutic intervention will score *equivalent* to a normal comparison group following the treatment
    - Often called 'normative comparisons'
  - Or, a researcher may hypothesize that caffeine intake is *not* related to levels of depression

Goal of Equivalence Tests

- The goal of equivalence tests is not to show that there is no relationship among variables (e.g., $\mu_1 = \mu_2$), only that any relationship that exists is too small to be considered meaningful (e.g., $-\delta < \mu_1 - \mu_2 < \delta$)
- The 'bounds' that define the upper and lower limits for an inconsequential relationship are termed the equivalence interval $(-\delta, \delta)$
  - In some cases a one-tailed equivalence test is conducted (e.g., inferiority tests), in which case there would only be an upper or lower bound, not an interval

Frequency of Equivalence-Based Hypothesis Tests

- Our lab conducted a review of the 2009 editions of three psychology journals to ascertain how frequent "equivalence based" research questions were being evaluated
  - Journal of Consulting and Clinical Psychology
  - Journal of Abnormal Psychology
  - Journal of Experimental Psychology-General
- For all three journals more than 50% of articles contained at least one equivalence-based hypothesis
  - The most frequent types of hypotheses were:
    - Group equivalence on demographics/other confounds
    - Primary Hypothesis(es)

Example: Equivalence-based Hypothesis

Personality and Individual Differences

Volume 74, February 2015, Pages 122-126

Comparing the psychosocial health of tattooed and non-tattooed women

Kathleen Thompson

Hypotheses

That tattooed women will be as psychosocially healthy as non-tattooed women as measured by scores on the Loyola Generativity Scale (LGS).

Frequency of Equivalence-Based Hypothesis Tests

- A more specific investigation of clinical studies was recently conducted

- Studies that compared psychological treatments and were published between 2000 and 2010 were included
  - 270 studies that compared two psychological treatments, psychological treatments to drug treatments, etc. were found
  - Of these studies, 154 specified no specific direction of effects, 91 hypothesized a difference between treatments, and 25 hypothesized that the treatments would be equivalent

Frequency of Equivalence-Based Hypothesis Tests

- Of the 25 studies that hypothesized equivalence, all used a difference based test for the comparison
  - Interestingly two studies used equivalence tests, but both incorrectly used them to investigate differences

- Further, approximately half of the studies that found no significant difference between treatments used "equivalence-based" language to summarize the findings
  - E.g.., "equivalence", "comparable", "equally effective"

What about at the U of M?

Reductions in Goal-Directed Cognition as a Consequence of Being the Target of Empathy

Jacquie D. Vorauer[1], Matthew Quesnel[1], and Sara L. St. Germain[1]

Personality and Social Psychology Bulletin
2016, Vol. 42(1) 130–141

DOI: 10.1177/0146167215617704
pspb.sagepub.com
SAGE

As preliminary analyses across the three studies revealed no evidence that sex moderated any effects of the mind-set manipulation, and sex did not vary according to experimental conditions ($Fs < 1$), it is not discussed further.

What about at the U of M?

Eur J Psychol Educ (2014) 29:175–194
DOI 10.1007/s10212-013-0193-2

The longitudinal effects of achievement goals and perceived control on university student achievement

Lia M. Daniels · Raymond P. Perry · Robert H. Stupnisky · Tara L. Stewart · Nancy E. G. Newall · Rodney A. Clifton

et al. 2008). In regards to achievement, we hypothesized that because this is a college sample, performance goals and primary control would positively and directly predict achievement, whereas mastery goals would have a non-significant direct effect. The existing research linking

What about at the U of M?

Canadian Journal of Experimental Psychology / Revue canadienne de psychologie expérimentale
2016, Vol. 70, No. 2, 93–98

1196-1961/16/$12.00 http://dx.doi.org/10.1037/cep0000082

The Production Effect in Recognition Memory: Weakening Strength Can Strengthen Distinctiveness

Glen E. Bodner
University of Calgary

Randall K. Jamieson and David T. Cormack
University of Manitoba

Dawn-Leah McDonald and Daniel M. Bernstein
Kwantlen Polytechnic University

implemented by Merritt, Cook, and Wang (2014). Three 2 (production: unproduced vs. produced) × 2 (design: mixed vs. pure) ANOVAs showed that the pure-list production effect was similar in size to the mixed-list production effect in each of the 20%, 50%, and 80% groups, $F(1, 50) = 1.62$, $p > .20$, $F(1, 50) = 2.77$, $p > .10$, and $F(1, 50) = 0.21$, $p > .60$. In summary, the pure- and mixed-list production effects were similar and no evidence for a distinctiveness influence was obtained.

Traditional and Equivalence-based Hypotheses

- Two Populations, Mean Difference
  - Traditional Nondirectional Null & Alternate Hypotheses
    - $H_o: \mu_1 = \mu_2$, $H_a: \mu_1 \neq \mu_2$
  - Equivalence Null & Alternate Hypotheses
    - Two One-sided Testing Procedure (TOST)
      - $H_{o1}: \mu_1 - \mu_2 \geq \delta$; $H_{o2}: \mu_1 - \mu_2 \leq -\delta$
      - $H_{a1}: \mu_1 - \mu_2 < \delta$; $H_{a2}: \mu_1 - \mu_2 > -\delta$
      - Rejection of H_{o1} implies that $\mu_1 - \mu_2 < \delta$, and rejection of H_{o2} implies that $\mu_1 - \mu_2 > -\delta$.
      - Thus, rejection of both null hypotheses implies that $\mu_1 - \mu_2$ falls within the bounds of $(-\delta, \delta)$ and the means are deemed equivalent

Can't a Traditional *t*-test be Used to Evaluate Equivalence?

- You cannot use non-rejection of the null hypothesis of a traditional difference-based *t*-test to evaluate equivalence because:
  - Theoretically, non-rejection of the null hypothesis does not prove the null to be true
  - Power is backward
    - Power increases as sample sizes decrease and error variances increase

Two One-sided Testing (TOST) for $\mu_1 - \mu_2$

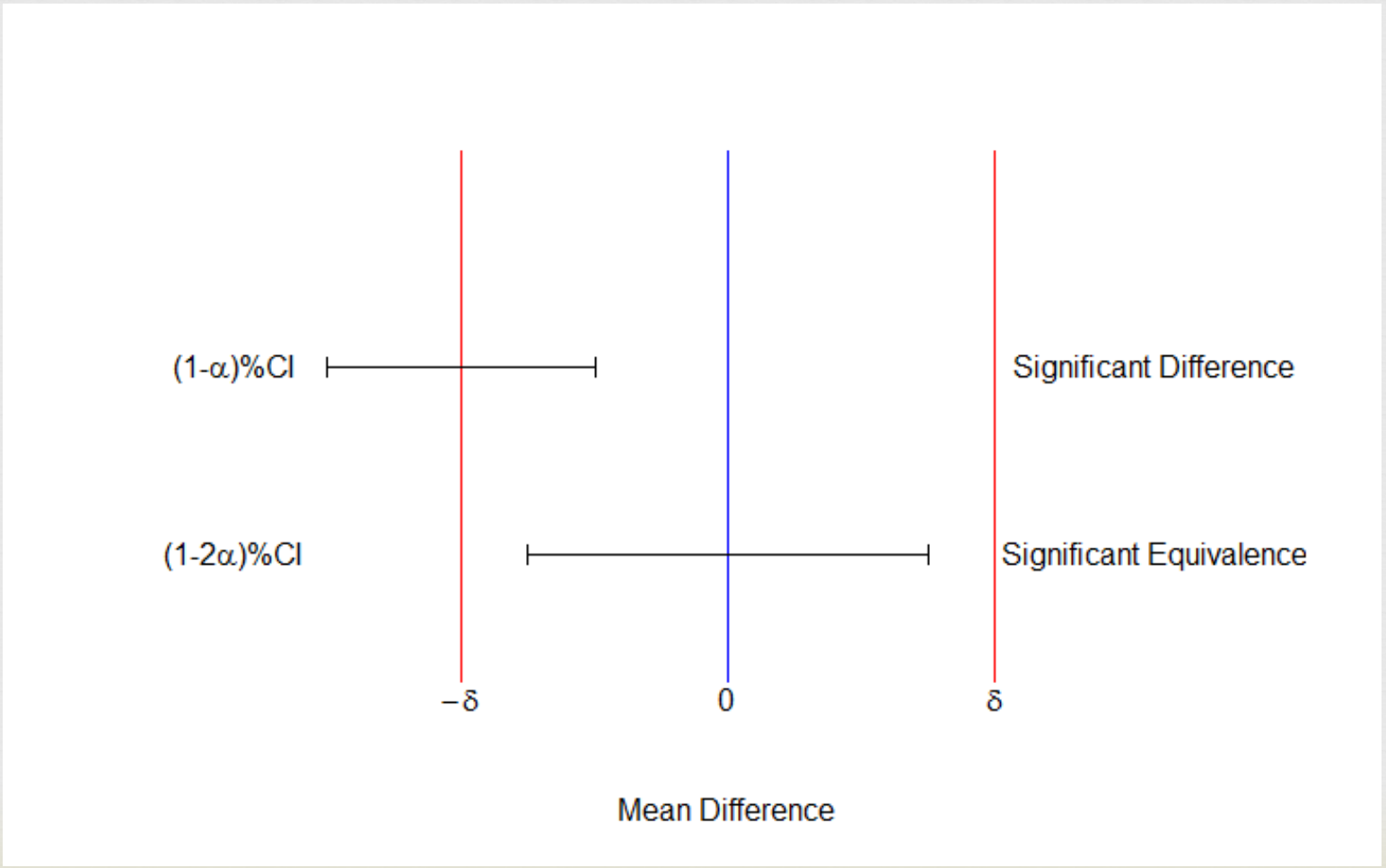
- $H_{o1}: \mu_1 - \mu_2 \leq -\delta$
  - H_{o1} is rejected if $t_1 \leq t_{\alpha,\, df=n_1+n_2-2}$
- $H_{o2}: \mu_1 - \mu_2 \geq \delta$
  - H_{o2} is rejected if $t_2 \geq t_{1-\alpha,\, df=n_1+n_2-2}$

$$t_1 = \frac{(\bar{X}_1 - \bar{X}_2) - \delta}{\sqrt{\frac{(n_1+n_2)[(n_1-1)s_1^2 + (n_2-1)s_2^2]}{n_1 n_2 (n_1+n_2-2)}}}$$

$$t_2 = \frac{(\bar{X}_1 - \bar{X}_2) - (-\delta)}{\sqrt{\frac{(n_1+n_2)[(n_1-1)s_1^2 + (n_2-1)s_2^2]}{n_1 n_2 (n_1+n_2-2)}}}$$

- Both H_{o1} and H_{o2} must be rejected in order to conclude population mean equivalence
- This is equivalent to testing if the (1-2α) CI falls within (-δ, δ)

Difference vs Equivalence: Confidence Intervals



Extensions of Equivalence Testing: Lack of Association

- One extension of equivalence testing is to the problem of demonstrating that two variables are *minimally related*

- For the same reason that a traditional *t* test cannot be used to evaluate the equivalence of two independent groups, a traditional correlation (or regression) statistic cannot be used to demonstrate a lack of association
  - Recall that the hypotheses are backward, and power would be maximized by decreasing N and increasing error variance

Lack of Association Tests

- The goal of a lack of association test is to demonstrate that any relationship between the variables is too small to be considered meaningful

- ρ^* is used to represent the smallest correlation between the variables that would be considered meaningful
  - $(-\rho^*, \rho^*)$ forms the equivalence interval

Lack of Association Test

- Following the logic of the TOST procedure, the composite null hypotheses, H_{o1}: $\rho \geq \rho^*$ and H_{o2}: $\rho \leq -\rho^*$, are rejected if $t_1 \leq t_{\alpha, N-2}$ and $t_2 \geq t_{1-\alpha, N-2}$, respectively, where:

$$t_1 = \frac{r - \rho^*}{\sqrt{\frac{1-r^2}{N-2}}} \qquad t_2 = \frac{r - (-\rho^*)}{\sqrt{\frac{1-r^2}{N-2}}}$$

We have also investigated versions of this test based on: 1) Fisher's *z* transformation, and 2) Resampling

Power Rates of Lack of Association Tests

	$N = 50$			$N = 100$			$N = 500$			$N = 1{,}000$		
ρ^*	eq_r	eq_fz	eq_rs	eq_r	eq_fz	eq_rs	eq_r	eq_fz	eq_rs	eq_r	eq_fz	eq_rs
$\rho = 0$												
.05	0	0	0	0	0	0	0	0	0	0	0	.003
.10	0	0	0	0	0	0	.425	.432	.434	.874	.874	.875
.15	0	0	0	0	0	0	.914	.917	.917	.998	.998	.999
.20	0	0	.008	.237	.262	.279	.995	.995	.995	1	1	1
.25	.044	.085	.137	.591	.621	.622	1	1	1	1	1	1
.3	.315	.365	.372	.820	.847	.849	1	1	1	1	1	1

TOST

Fisher's *z*

Resampling

Lack of Association Tests

- Why is it hard to find a 'lack of association' with small n/ρ*?

ρ = 0

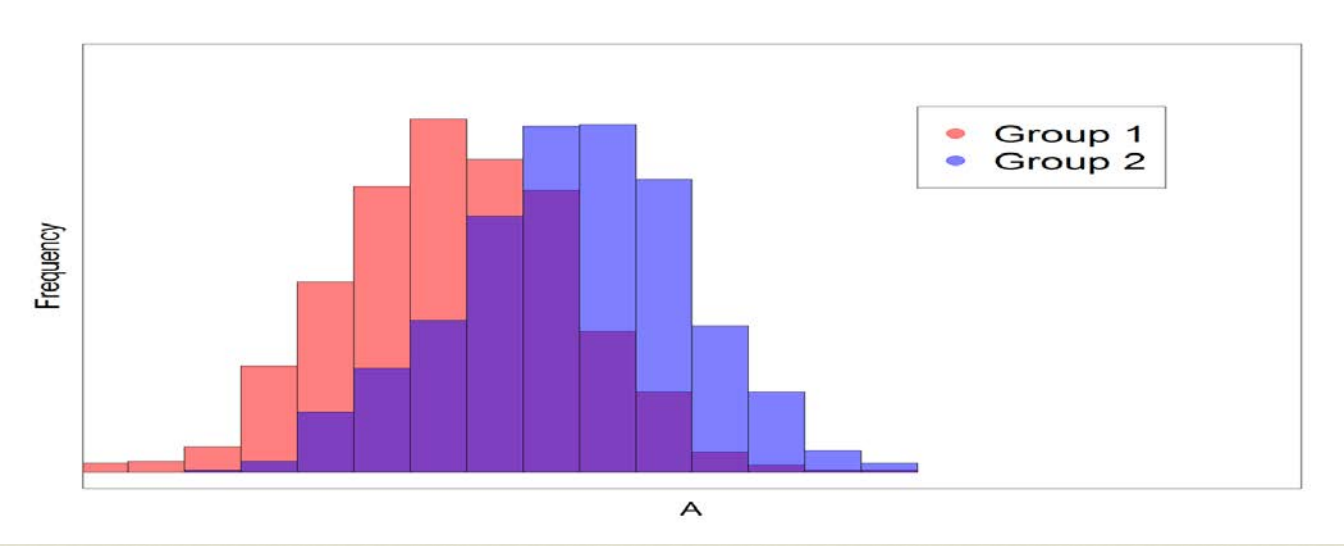
	Sample correlation (r) magnitude (absolute value)									
N	>.9	>.8	>.7	>.6	>.5	>.4	>.3	>.2	>.1	>0
10	.001	.007	.025	.067	.140	.252	.398	.578	.780	1
15	0	<.001	.005	.019	.060	.139	.273	.465	.720	1
20	0	0	.001	.005	.025	.082	.199	.400	.687	1
25	0	0	<.001	.002	.012	.046	.145	.334	.628	1
50	0	0	0	0	<.001	.004	.036	.163	.492	1
100	0	0	0	0	0	0	.003	.048	.323	1
200	0	0	0	0	0	0	0	.005	.159	1

What is an appropriate equivalence interval?

- The "Elephant in the Room" when it comes to many discussions of equivalence testing
  - Essentially, a researcher is asked to determine *the smallest effect that is of practical importance* within the nature of the study
  - Our lab has started investigating what represents the smallest meaningful relationship among variables in common research settings (almost no prior research on the topic)

Smallest Meaningful Difference in Central Tendencies

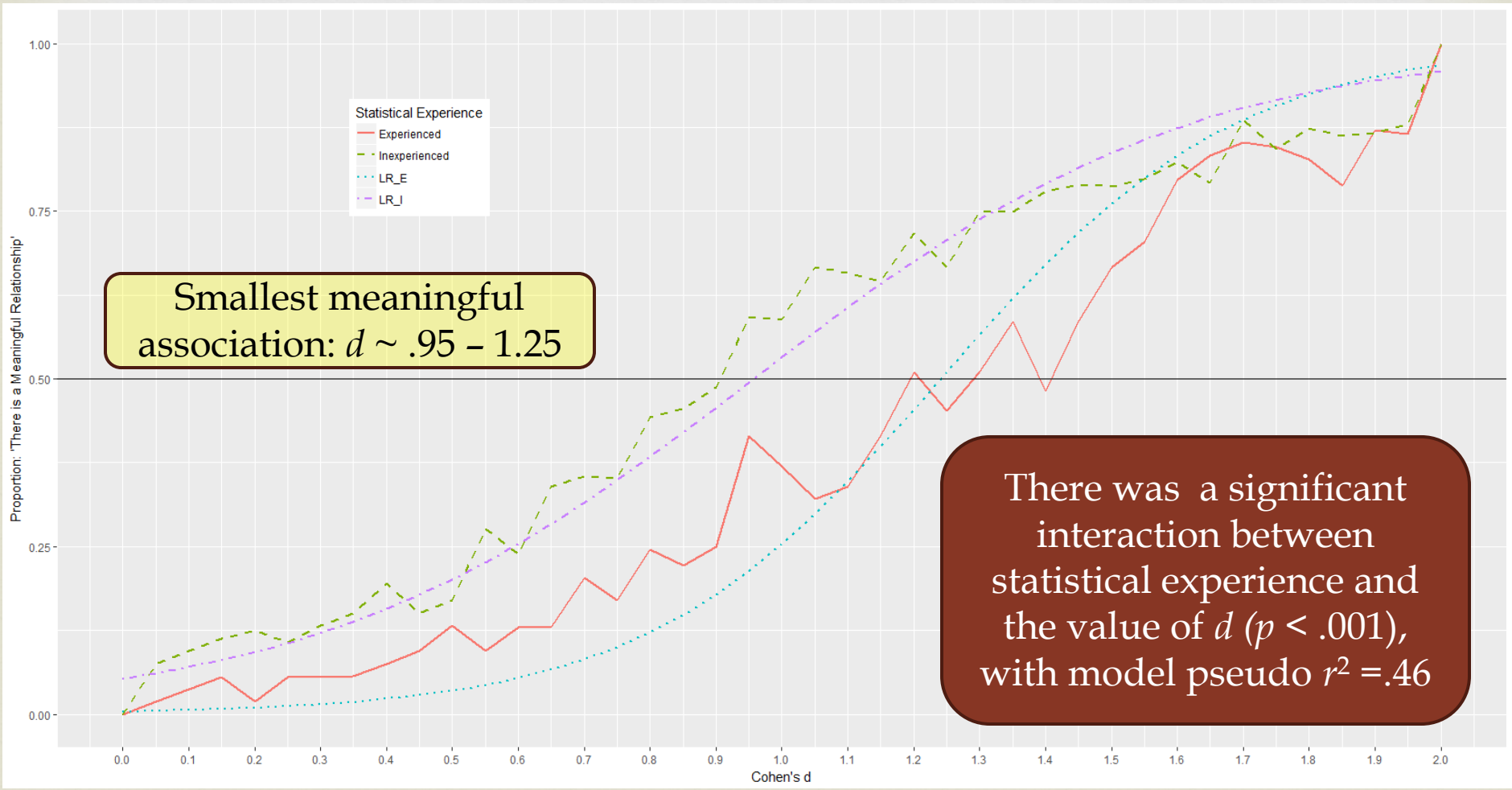
- Overlapping histograms displayed two distributions that were separated by a population Cohen's *d* ranging from 0.00 to 2.00, in .05 increments
  - For each value of Cohen's *d*, five plots were generated and subjects saw only one randomly chosen plot



Smallest Meaningful Difference in Central Tendencies

- Participants were asked to indicate whether the difference in the distributions was meaningful
- Training for interpreting the plots was provided
- Participants were separated by level of statistical experience (< 3 semester courses in university statistics, 3+ semester courses in university statistics)
- Based on past research, the value of Cohen's *d* where 50% of participants assert that the relationship is meaningful was used to represent the smallest meaningful association

Logistic Regression Results – Mean Difference

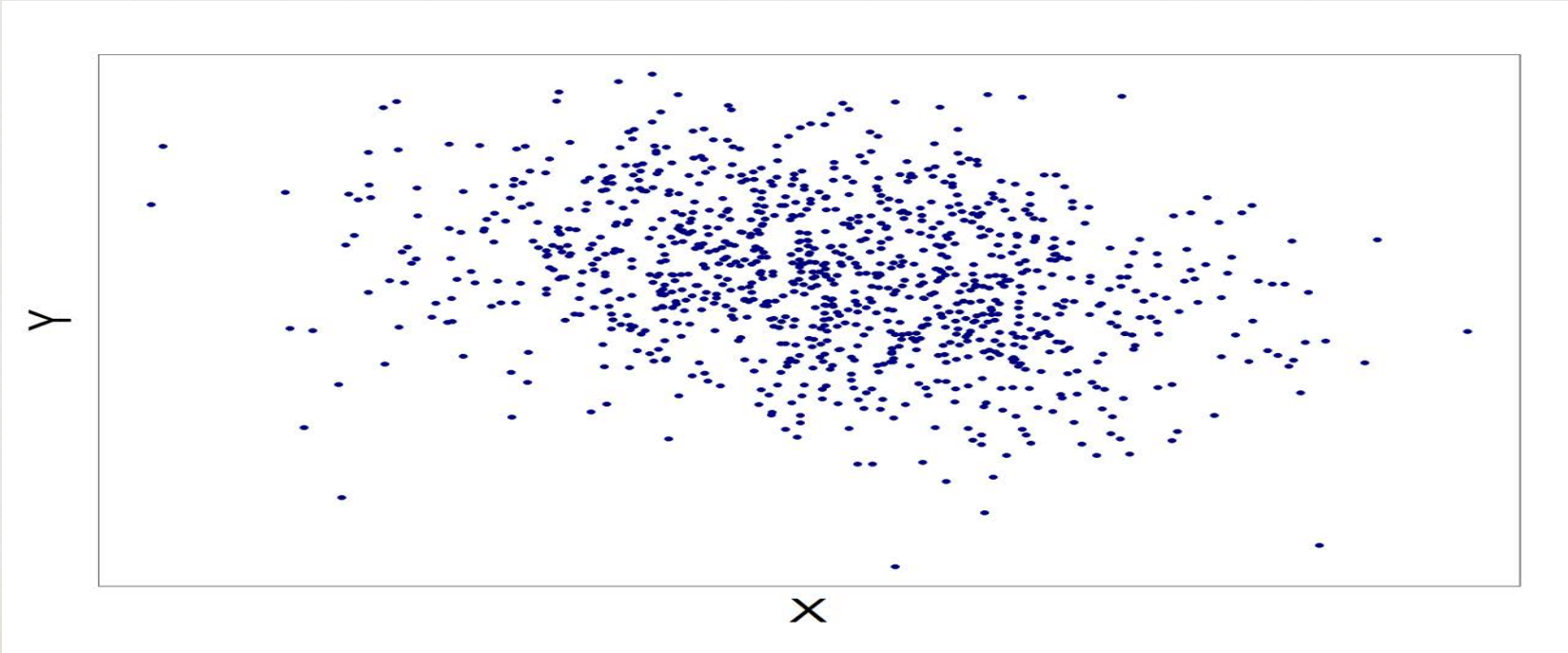


Smallest meaningful association: $d \sim .95 - 1.25$

There was a significant interaction between statistical experience and the value of d ($p < .001$), with model pseudo $r^2 = .46$

Smallest Meaningful Association among Variables

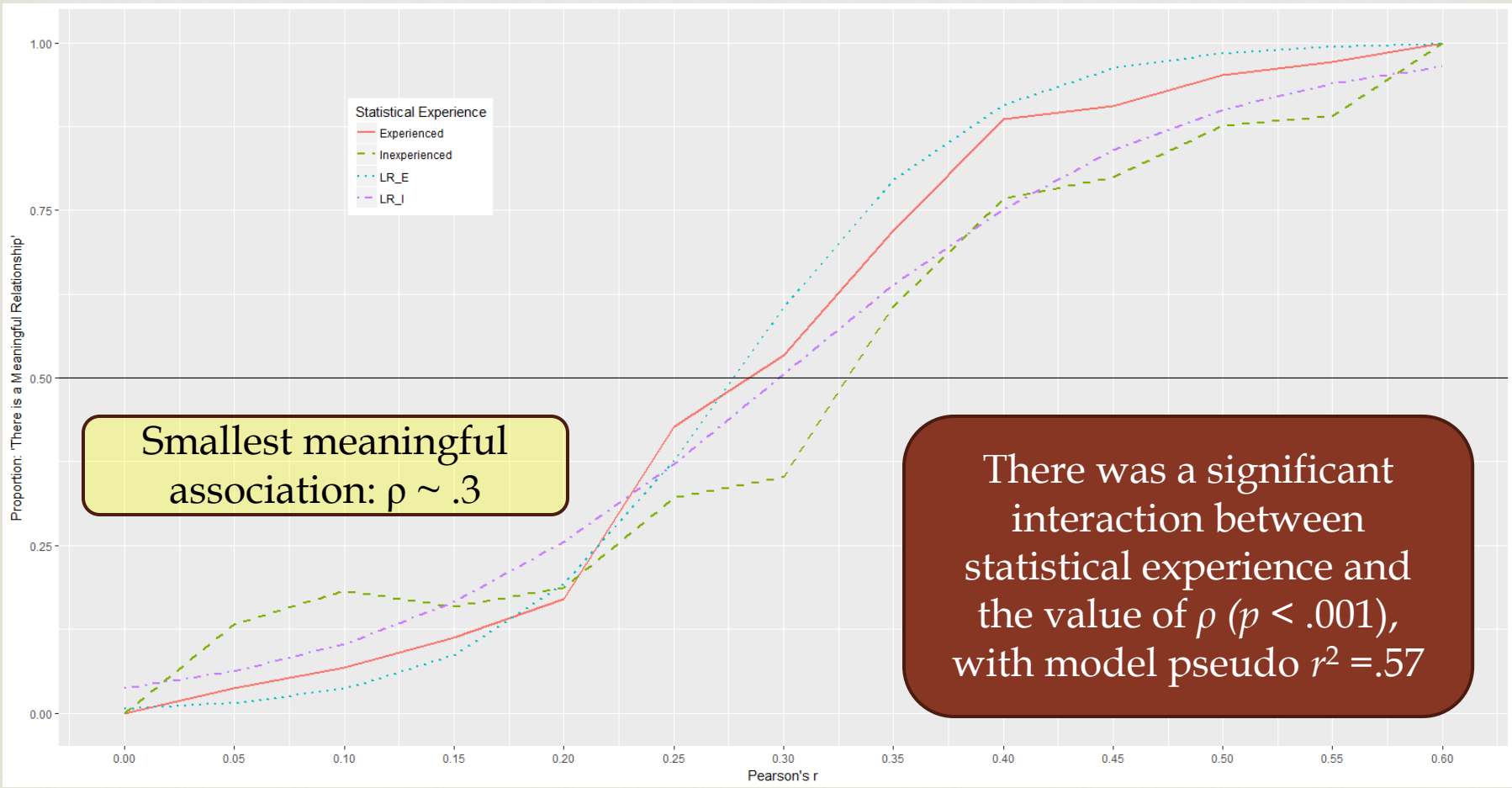
- Scatterplots displayed population correlations ranging from ρ = - .60 to ρ = .60 in .05 increments
  - For each value of ρ, five plots were generated and subjects saw only one randomly chosen plot



Smallest Meaningful Association among Variables

- Participants were asked to indicate whether the association was meaningful
- Training for interpreting the plots was provided
- Participants were separated by level of statistical experience (< 3 semester courses in university statistics, 3+ semester courses in university statistics)
- Again, the value of ρ where 50% of participants assert that the relationship is meaningful was used to represents the smallest meaningful association among the variables

Logistic Regression Results – Correlation



Smallest meaningful association: ρ ~ .3

There was a significant interaction between statistical experience and the value of ρ ($p < .001$), with model pseudo r^2 =.57

Conclusions – Smallest Meaningful Association

- The minimally important relationship in a correlation setting (ρ = .30 to ρ = .35) was moderately higher than what Cohen suggested as the lower bound for a "small" correlation (ρ = 0.10)
- The minimum meaningful difference in central tendencies (d = .95 to d = 1.20) was much larger than what Cohen suggested as the lower bound for a "small" standardized mean difference (d = 0.20)
- We re-ran the study asking if "the groups were equivalent" or "there was a negligible association" and found essentially the same results

Conclusions – Smallest Meaningful Association

- It is hoped that this research will help researchers better interpret the magnitude of effect sizes and set appropriate boundaries in equivalence testing
  - Current research is exploring what aspects of the figures participants focus on when making decisions regarding the meaningfulness of the association
    - Eye Tracking
  - Another current study allows participants to manipulate the visualizations until the smallest meaningful association is represented
    - Will this produce different results?

Extensions of Equivalence Testing

- Our lab has explored many proposed extensions of equivalence testing, including:
  - Tests of Substantial Mediation (Mara)
  - Tests of the Equivalence of Correlation or Regression Coefficients across Groups (Counsell)
  - Negligible Interaction Tests (Counsell, Jabbari)
  - Multiplicity Issues in Equivalence Testing (Davidson)
  - Equivalence Tests for Longitudinal Data (Ng, Mara)
  - Equivalence Tests for Categorical Data (Shiskina)
  - Bayesian approaches to Equivalence Testing (Counsell, Hoyda)
  - Equivalence-based Homogeneity of Variance Test (Mara, Kim)
  - Equivalence Tests for Measurement Invariance (Counsell)

Current Approaches to Testing for Variance Equality

- Difference-based tests
  - E.g., Levene's (1960) test for homogeneity of variances (HOV)
    - An ANOVA is conducted on the absolute value of the deviations from the group means:
      - $Z_{ij} = |X_{ij} - \bar{X}_j|$
- Brown-Forsythe (1974): $Z_{ij} = |X_{ij} - Mdn_j|$

Levene's HOV Test

- $H_o: \sigma_1^2 = \sigma_2^2$
  - HOV is concluded when H_o is NOT rejected
- Since the research hypothesis deals with variance equality, H_a, not H_o, should be aligned with the research hypothesis

Applying Equivalence Testing to Homogeneity of Variance Tests

- Wellek's (2003) one-way equivalence test is used for detecting the equivalence of multiple population means
- Borrowing logic from Wellek's one-way equivalence test, and Levene's HOV test:

$$H_0 : \Psi^{2^*} \geq \varepsilon^2$$

- Ψ^{2*} is an estimate of variance inequality (i.e., a modified ANOVA on the $Z_{ij} = |X_{ij} - \bar{X}_j|$)

$$H_a : \Psi^{2^*} < \varepsilon^2$$

- ε^2 represents the smallest difference in the variances that is meaningful (same metric as Ψ^{2*})

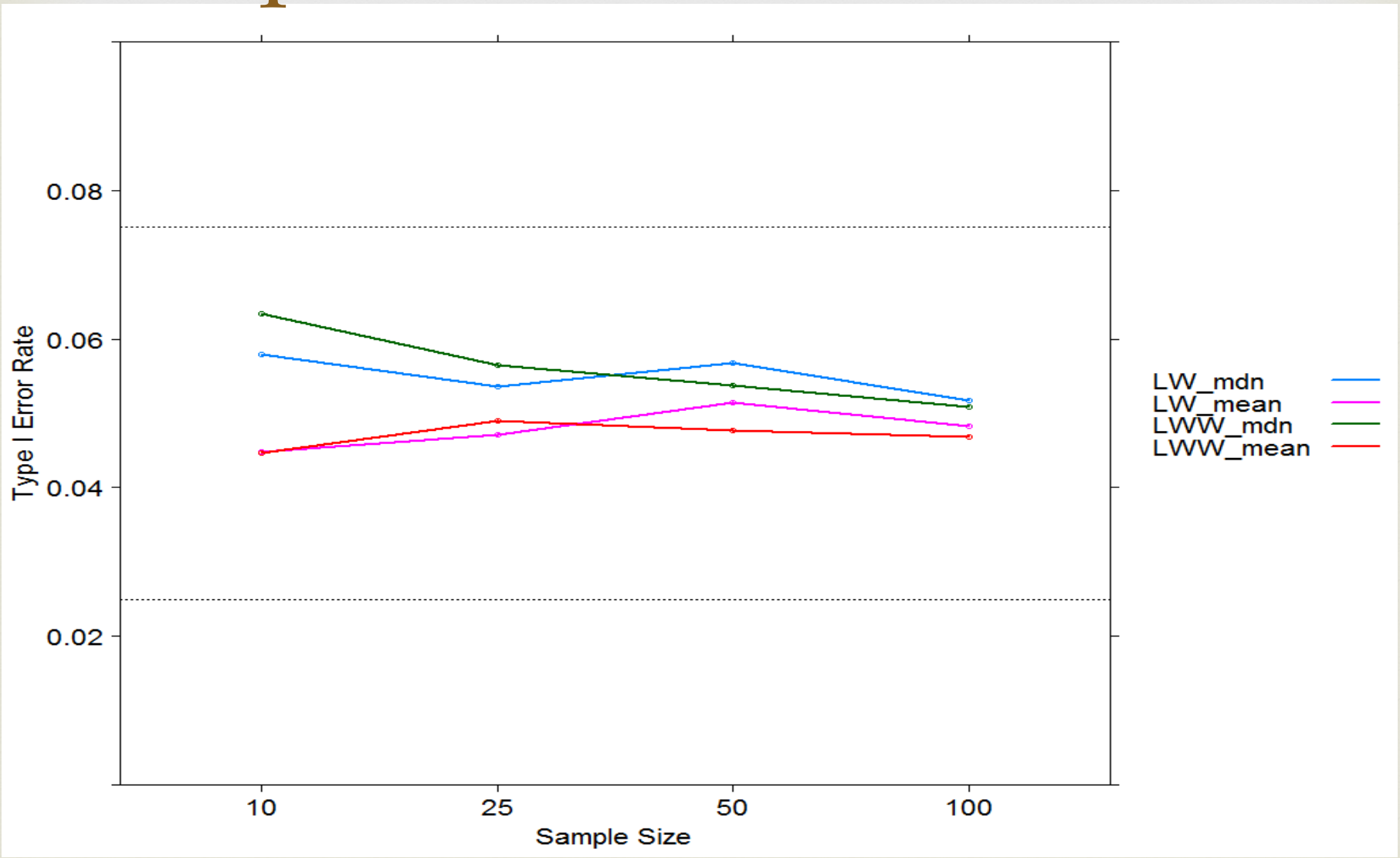
Monte Carlo Study

- 4 equivalence tests:
  - Levene-Wellek using $Z_{ij} = \left|X_{ij} - \bar{X}_j\right|$ (LW_mean)
  - Levene-Wellek using $Z_{ij} = \left|X_{ij} - Mdn_j\right|$ (LW_mdn)
  - Welch-adjusted versions of each of these tests (LWW_mean, LWW_mdn)
- Compared to 4 non-equivalence versions:
  - Original Levene (Lev_mean)
  - Brown-Forsythe (Lev_mdn)
  - Welch-adjusted versions of each of these tests (LevW_mean, LevW_mdn)

Monte Carlo Study

- Outcomes:
  - Type I error control
  - Probability of detecting equivalence
    - Equivalence Test: Reject H_0
    - Traditional Levene Test: Don't Reject H_0
- Manipulated variables:
  - Group sample sizes
  - Level of variance equality
  - # of groups = 2 & 4
  - Equivalence Interval (ε) =.50 & .25
  - α = .05

Mean Type I Error Rates: Equivalence-based Tests



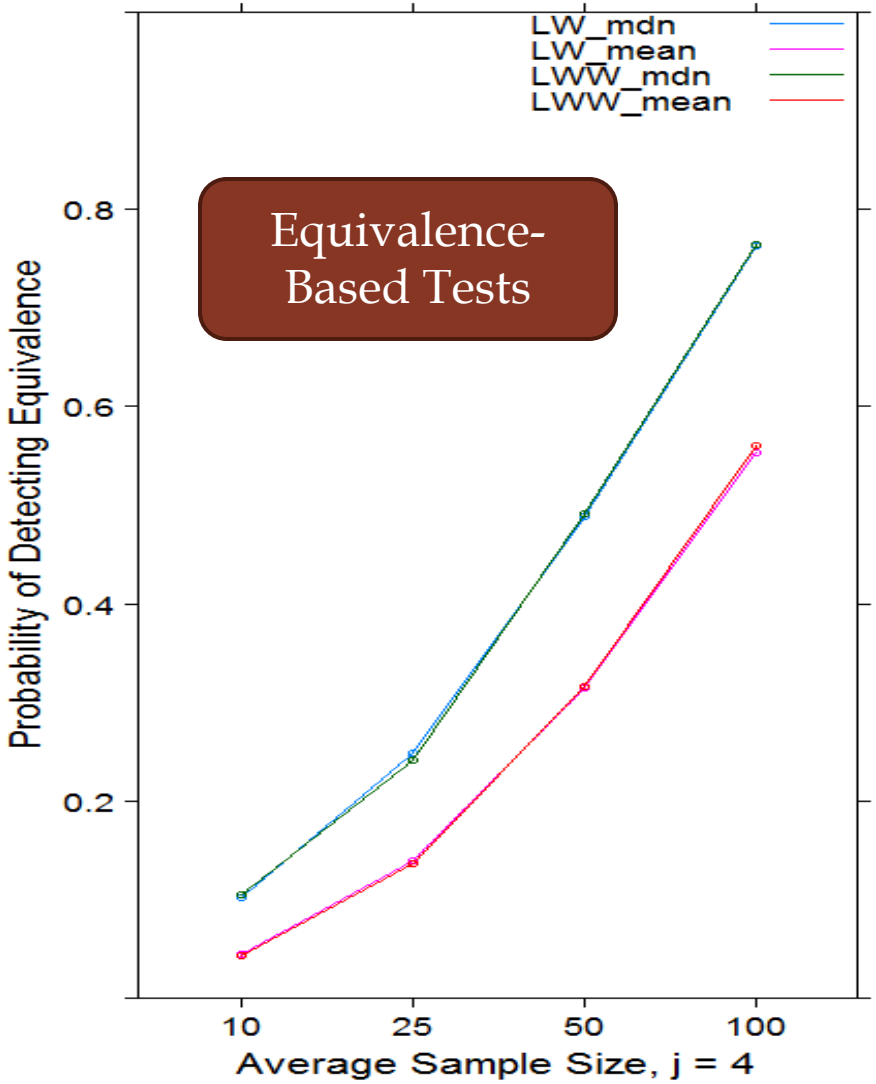
Type I Error Rates

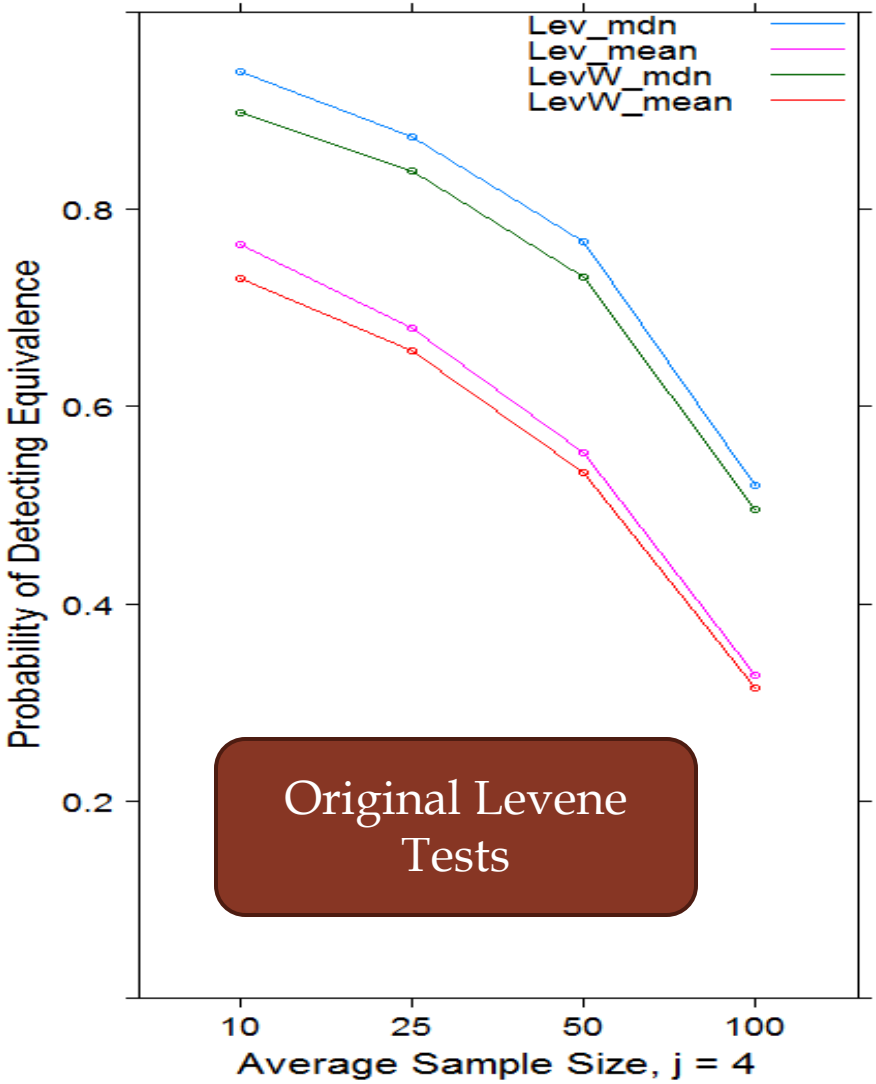
Test	Min. Empirical Type I Error Rate	Max. Empirical Type I Error Rate	# of Times Type I Error Rate Exceeded the Bounds of .025-.075
Levene-Wellek mean	.0200	.1113	12
Levene-Wellek median	.0223	.0886	6
LWW mean	.0182	.0859	9
LWW median	.0237	.1014	2

Issues in only 2/96 conditions

Probability of Declaring Equivalence (4 Groups)

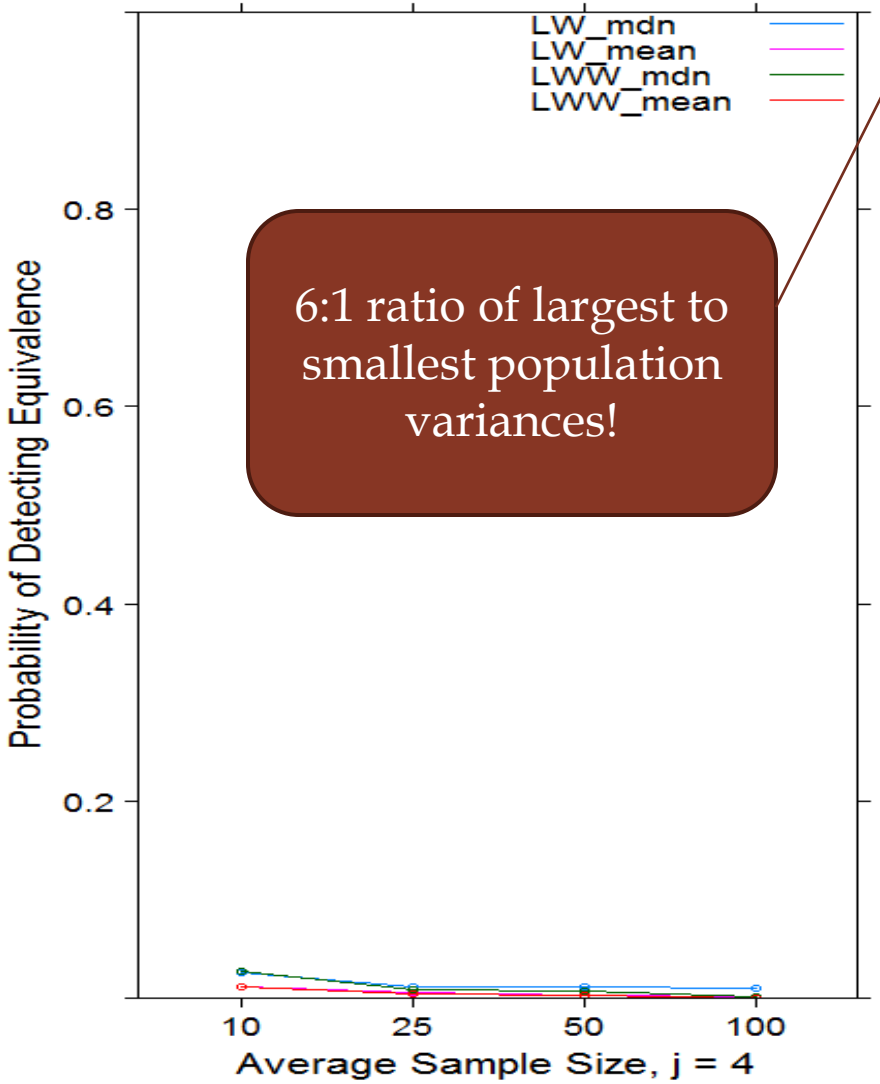
$\varepsilon = .50$, $\sigma^2 = 1, 1.33, 1.66, 2$ $(\Psi^2 < \varepsilon^2)$



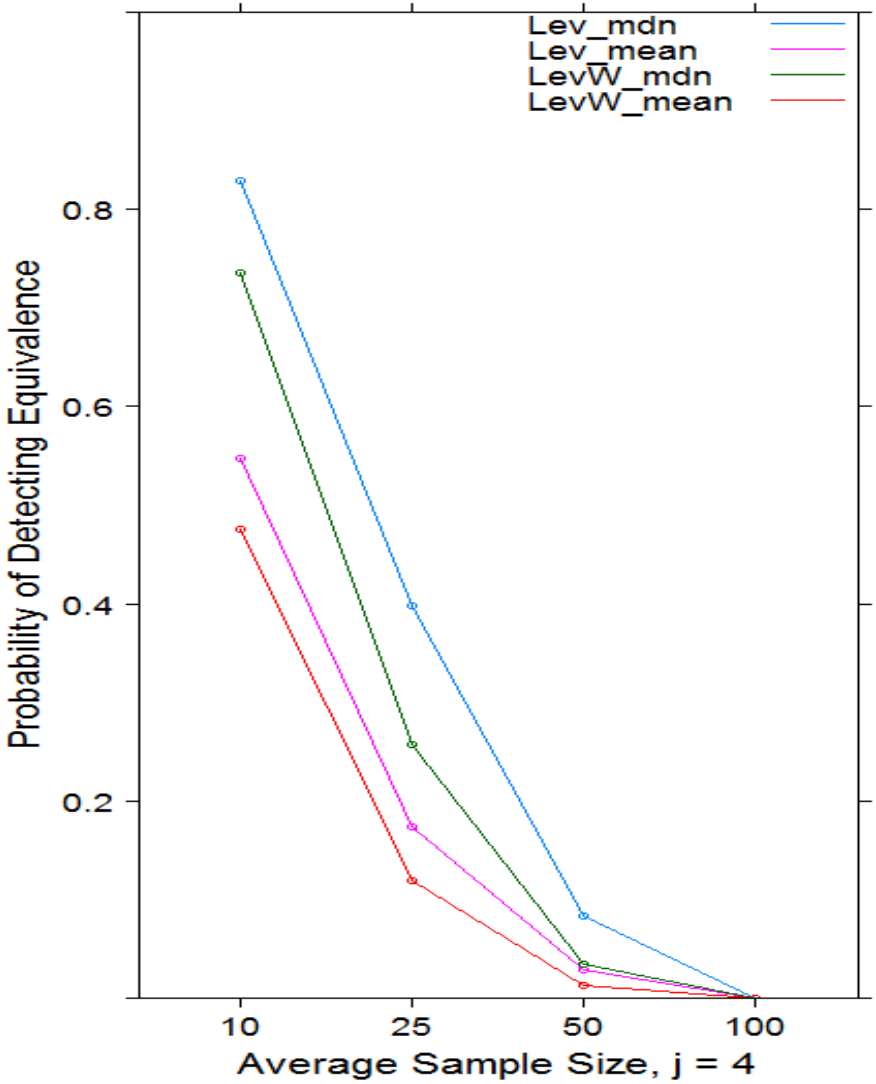


Average Probability of Declaring Equivalence

$\varepsilon = .50$, $\sigma^2 = 1, 3, 4, 6$ $(\Psi^2 > \varepsilon^2)$



6:1 ratio of largest to smallest population variances!



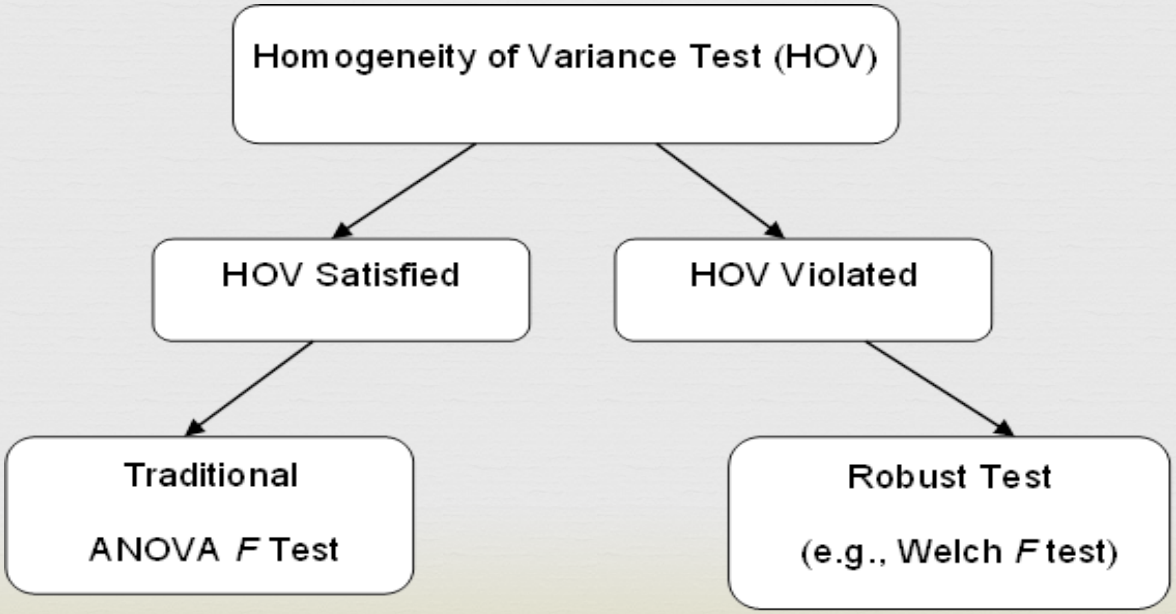
Conclusion - HOV

- Traditional HOV tests address the problem from the wrong perspective
  - Levene difference-based tests attempt to NOT REJECT H_0: $\sigma_1^2 = \sigma_2^2$

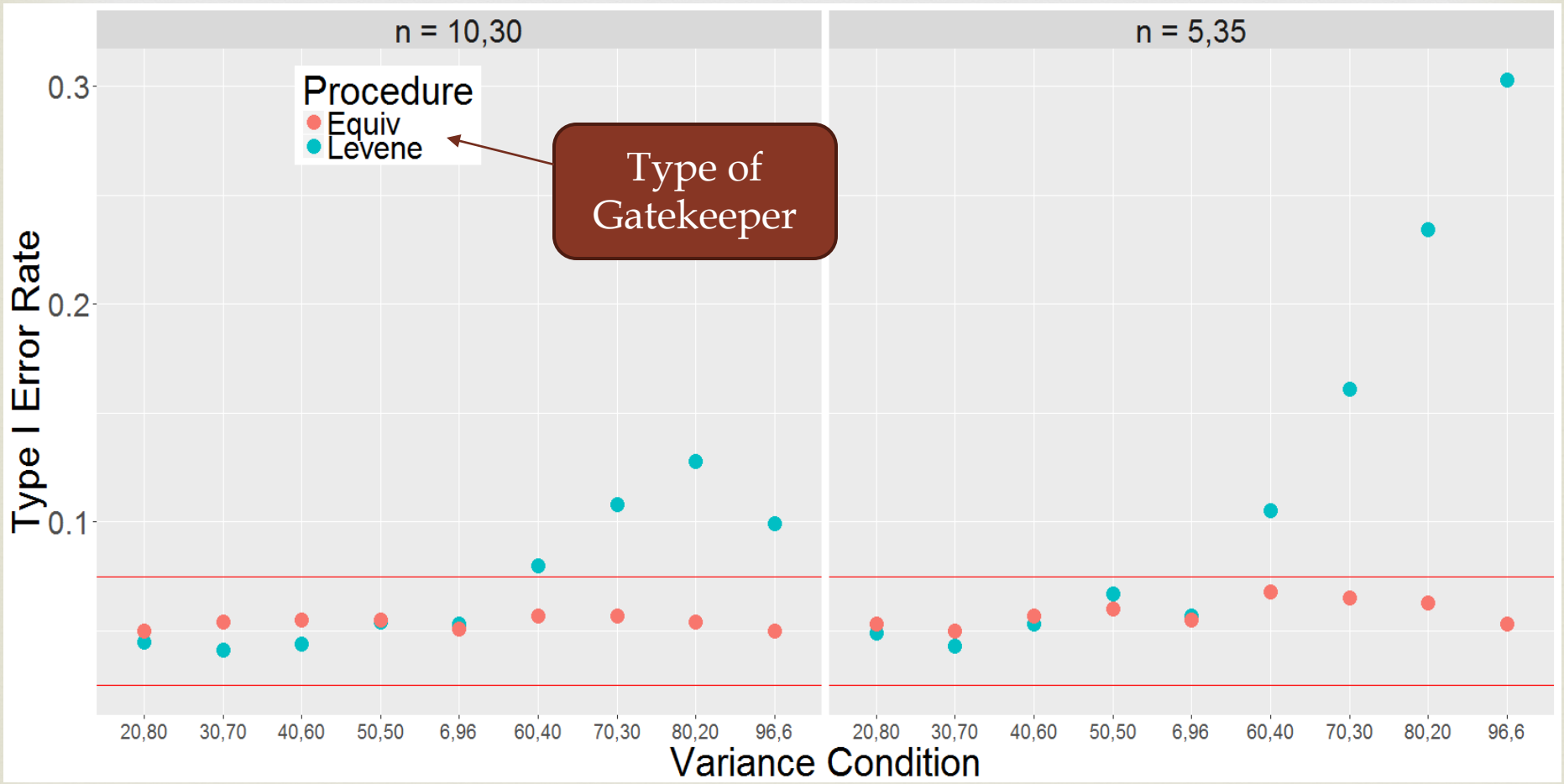
- The proposed equivalence-based tests correctly address the research question: "Are the population variances equivalent?"
  - Power increases with sample size and large differences in variances are not declared equivalent with small N

Extension: Equivalence-based HOV Tests

- Previous research has found that Levene-type tests are not effective gatekeepers for deciding between traditional and robust tests
- But what about an equivalence-based HOV test?



Type I error Rates With Gatekeepers

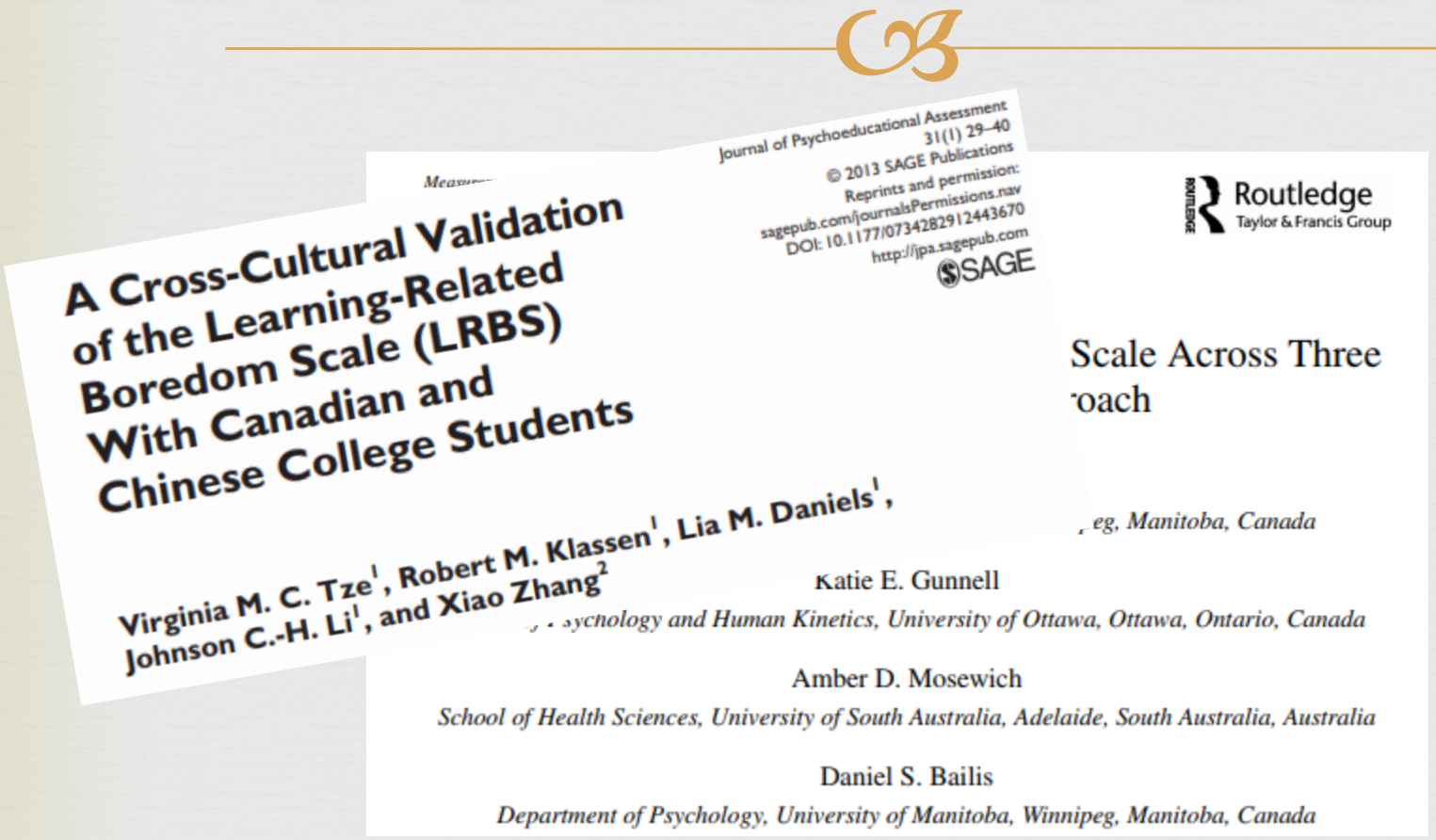


Measurement Invariance (MI)

- Researchers often seek to compare independent groups on a construct of interest
  - For example, do males and females score similarly/differently on depression?

- When discussing group differences on a construct, it is important to ensure that these differences are indeed a function of the group membership, rather than the manner in which the construct is measured
  - For example, maybe the items on a depression scale are interpreted differently by males and females

MI at the U of M



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A Cross-Cultural Validation of the Learning-Related Boredom Scale (LRBS) With Canadian and Chinese College Students

Virginia M. C. Tze[1], Robert M. Klassen[1], Lia M. Daniels[1], Johnson C.-H. Li[1], and Xiao Zhang[2]

Routledge
Taylor & Francis Group

Scale Across Three
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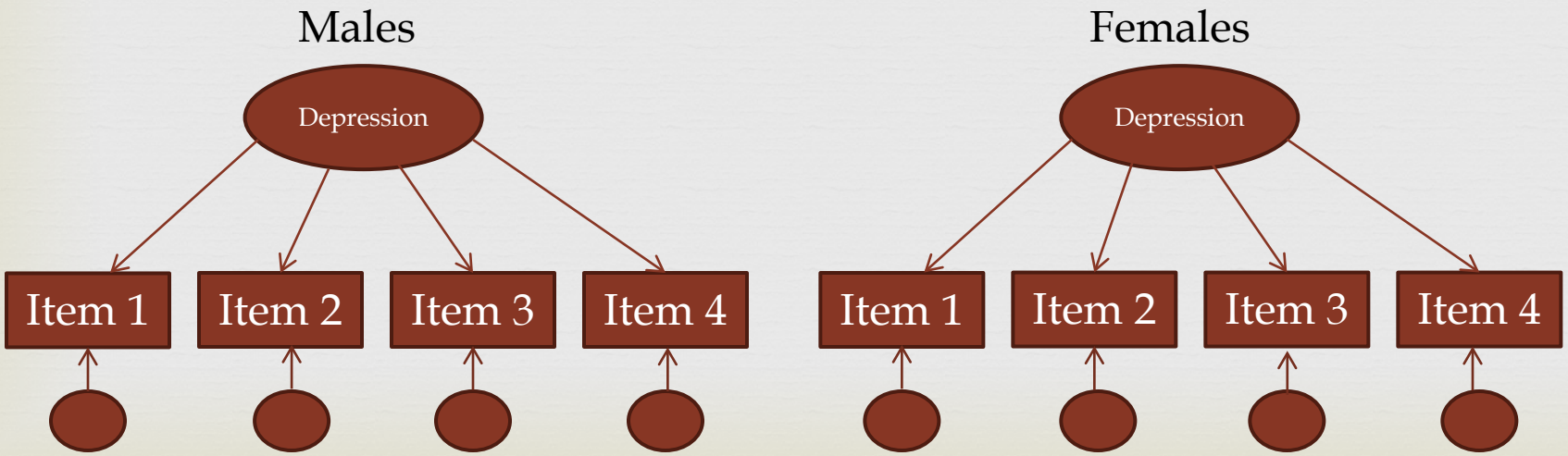
Assessing MI

- There are multiple levels of MI
  - **Configural Invariance: same factor structure**
  - Metric (Weak) Invariance: factor loadings are equal
  - Scalar (Strong) Invariance: intercepts are equal
  - Strict Invariance: error variances are equal



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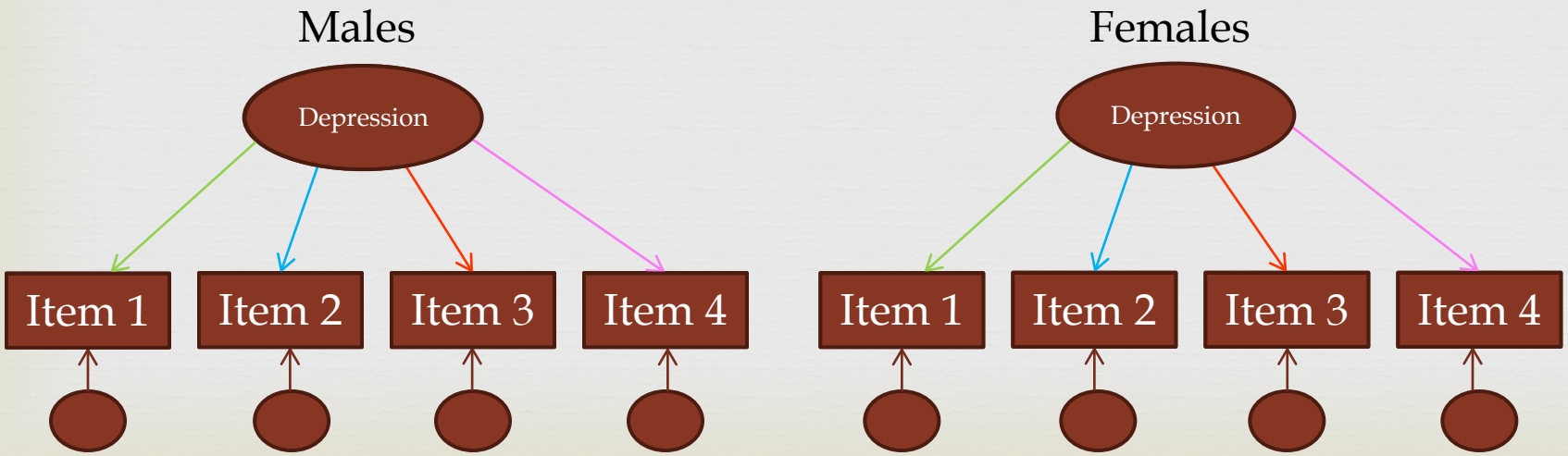

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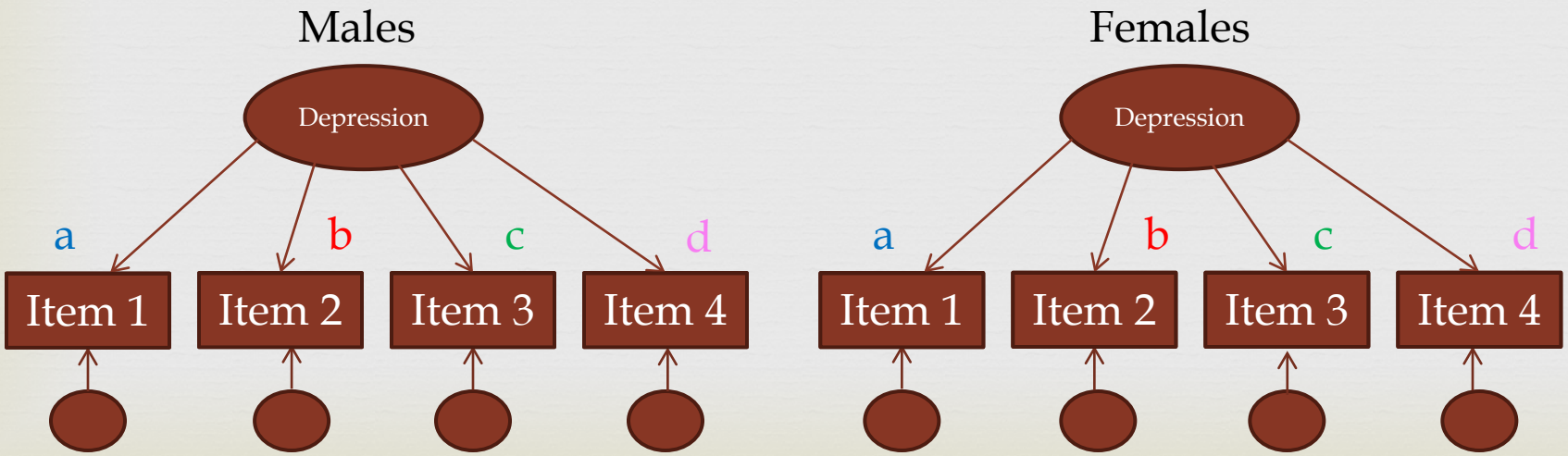

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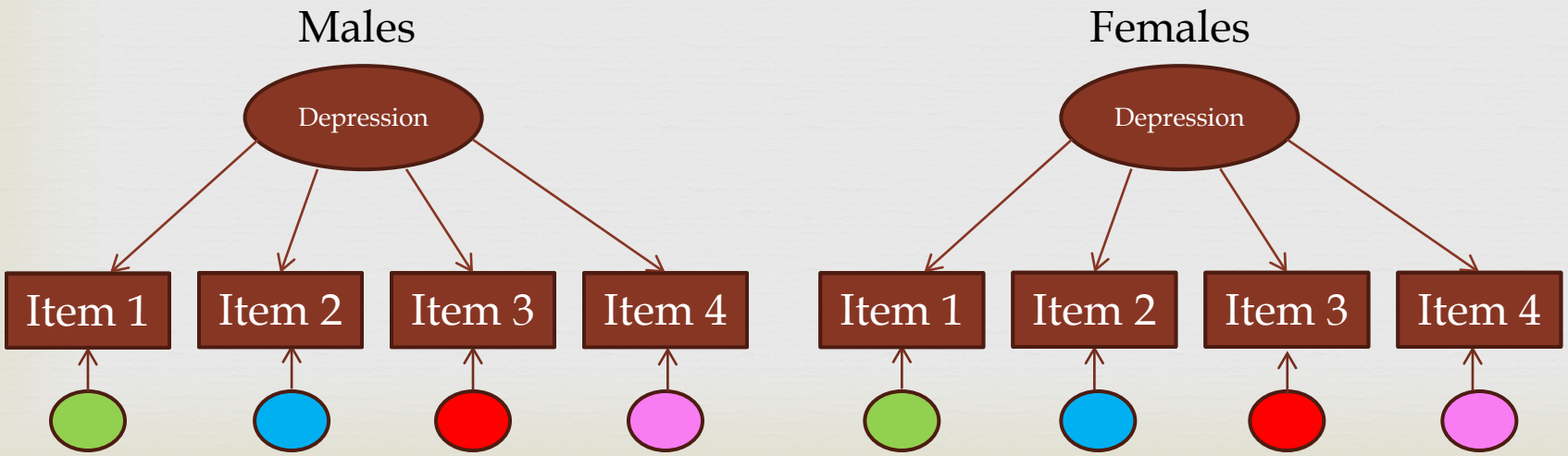
  - Scalar (Strong) Invariance: intercepts are equal
  - **Strict Invariance: error variances are equal**



Traditional Approaches to MI

- Nonsignificant χ^2 difference test
  - Nested model comparisons ($\chi^2_{\text{more constrained}} - \chi^2_{\text{less constrained}}$)
  - Example (metric invariance): Is the χ^2 fit statistic significantly smaller if we fix the factor loadings to be equal in males and females than if we let males and females have unique loadings?

- Using change in fit indices (ΔGOF)
  - ΔCFI (Comparative Fit Index)
  - ΔMNCI (Macdonald's Noncentrality Index)
  - ΔRMSEA (Root Mean Square Error of Approximation)

Issues with Traditional Approaches to MI

- The χ^2 difference test approach has several limitations:
  - "Accepting" the null hypothesis
  - Unrealistic to expect *zero* difference in any parameters between groups
  - Power to find invariance is highest when sample sizes are small
- Fit indices are descriptive in nature and there is much debate about appropriate ΔGOF cut-offs

Equivalence Tests for MI

- Yuan and Chan (2016) proposed using equivalence testing principles to evaluate MI
- Equivalence testing null hypotheses:
  - $F_{ml0} > \varepsilon$ at the configural stage
  - $F_{bc0} - F_{b0} > \varepsilon$ for all subsequent stages

where F_{ml0} is the population fit function, and $F_{bc0} - F_{b0}$ is the difference in fit functions of two nested models where *b* indexes the baseline model and *bc* indexes the baseline model with constraints, and ε is the largest tolerable amount of model misspecification

What is ε?

- As discussed earlier, one of the biggest challenges with equivalence testing is setting an appropriate equivalence interval
- Yuan and Chan (2016) relate ε to the RMSEA

$$\varepsilon = \frac{df(RMSEA^2)}{K}$$

- Where *df* is the model degrees of freedom at the configural stage or difference in *df* when comparing nested models, *K* is the number of groups
- This value is rescaled into a noncentrality parameter (ncp) for use in calculating a noncentral χ^2 statistic

$$\delta = (N-K)\varepsilon$$

Yuan and Chan's Recommended Adjustment

- Despite outlining this test statistic with conventional RMSEA values used for calculating ε, Yuan and Chan argue that power based on these values is too low

- They provide functions for an adjusted ε/RMSEA but provide little theoretical or empirical justification for the adjusted test's performance

Simulation Study

- We evaluated the power and Type I error control of Yuan and Chan's outlined equivalence testing method (EQ) and their recommended modification using adjusted RMSEA values (EQ-A)

- The equivalence tests' performance was compared to using a nonsignificant χ^2 difference test and ΔGOF
  - For simplicity the results for the ΔGOF method are not presented, but a brief conclusion regarding the method is provided

Conditions

- Measurement Model
  - 2 factors with either 4 or 8 indicators each
- Sample Size
  - 100, 250, 500, 1000, or 2000 per group
- Equivalence Bound (ε)
  - Based on RMSEAs of .05, .08, or .10
- Factor Loadings
  - .5, .7, .9

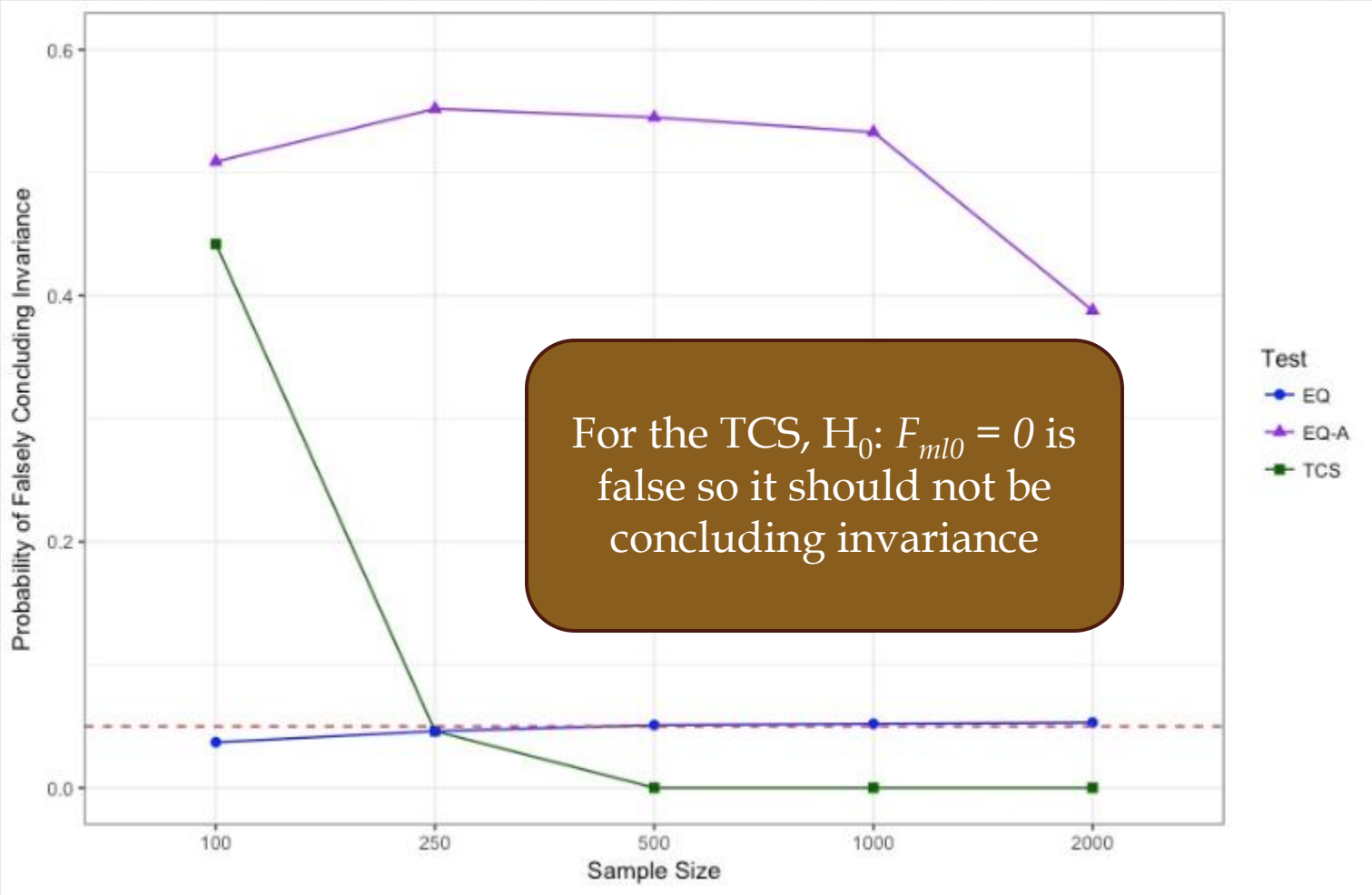
Type I Error and Power

- To evaluate Type I error rates, the population model misspecification was created such that $F_{ml0} = \varepsilon$ in each group at the configural stage and $F_{bc0} - F_{b0} = \varepsilon$ at all other stages
  - Error covariances were added to differing observed variables in each group at the configural stage to invoke lack of fit
  - At later stages, either one parameter differed or 25% of parameters differed (e.g., loading, intercept, error variance)

- To evaluate power we tested a condition where the groups' population models were identical and one where there were differences smaller than ε

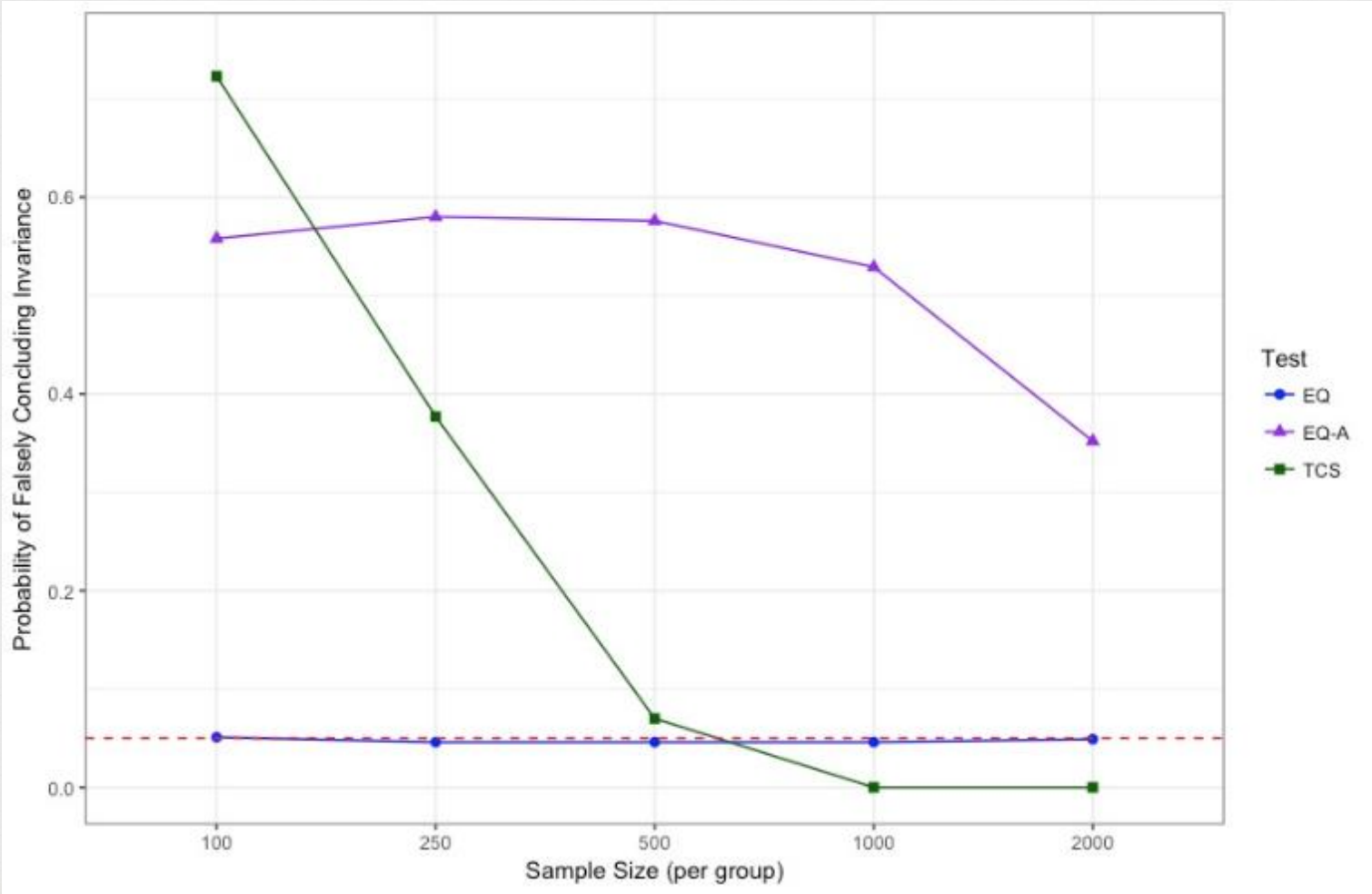
Type I Error Results

Configural Invariance: RMSEA = .08, 4 indicator Model



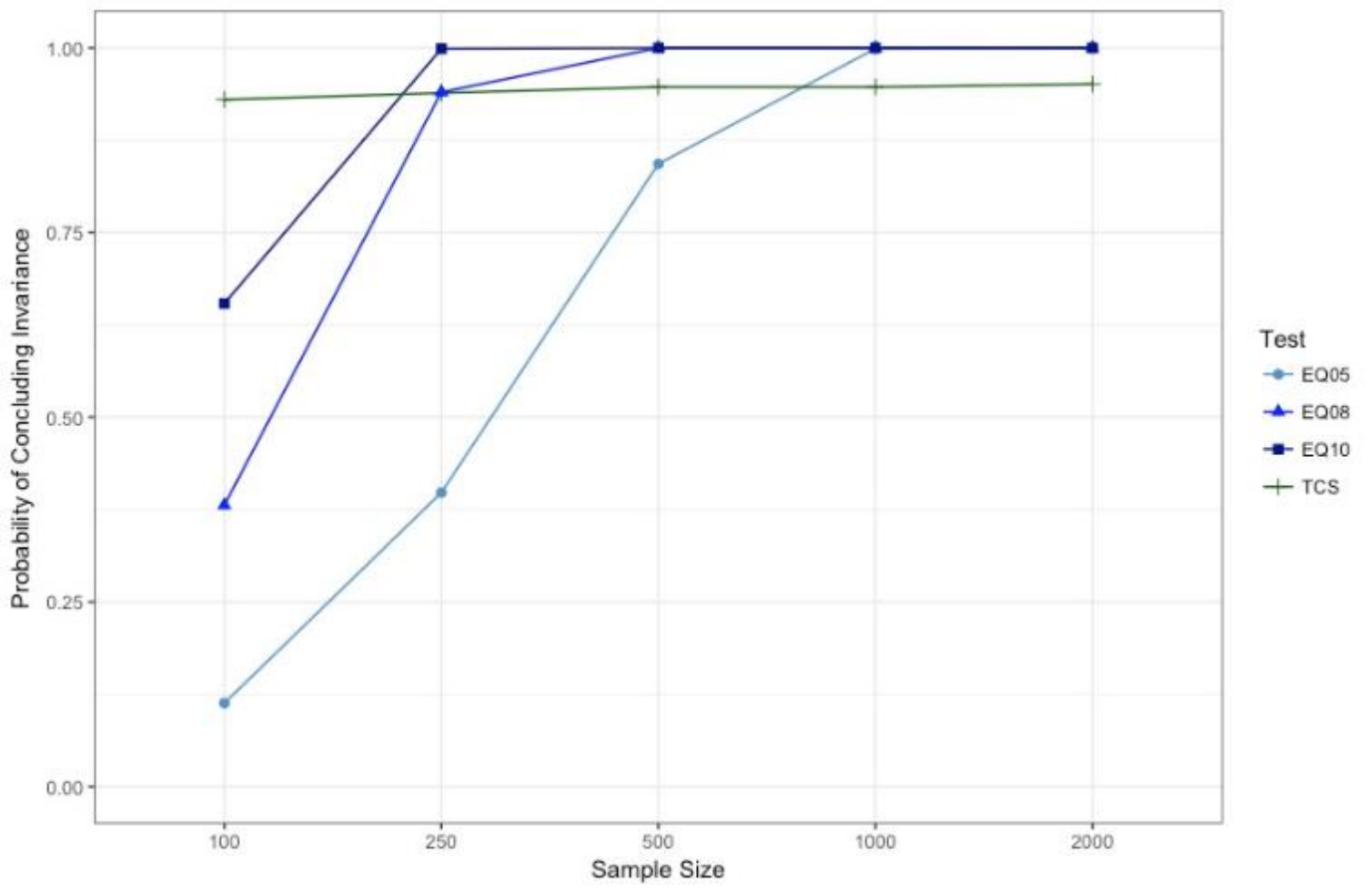
Type I Error Results

Metric Invariance: RMSEA = .08, 4 indicator Model



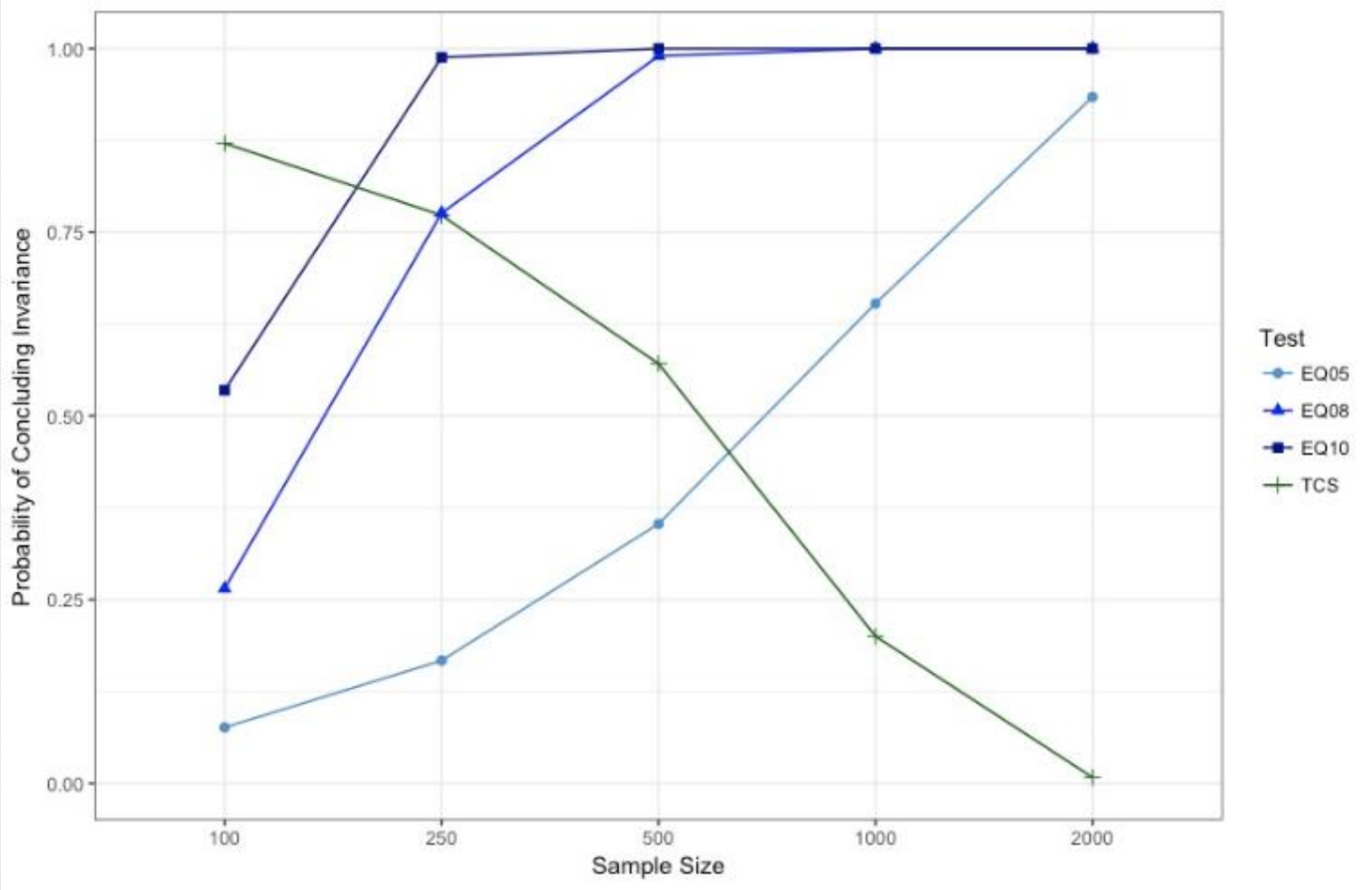
Power Results

Configural Invariance: 4 indicators, equal models



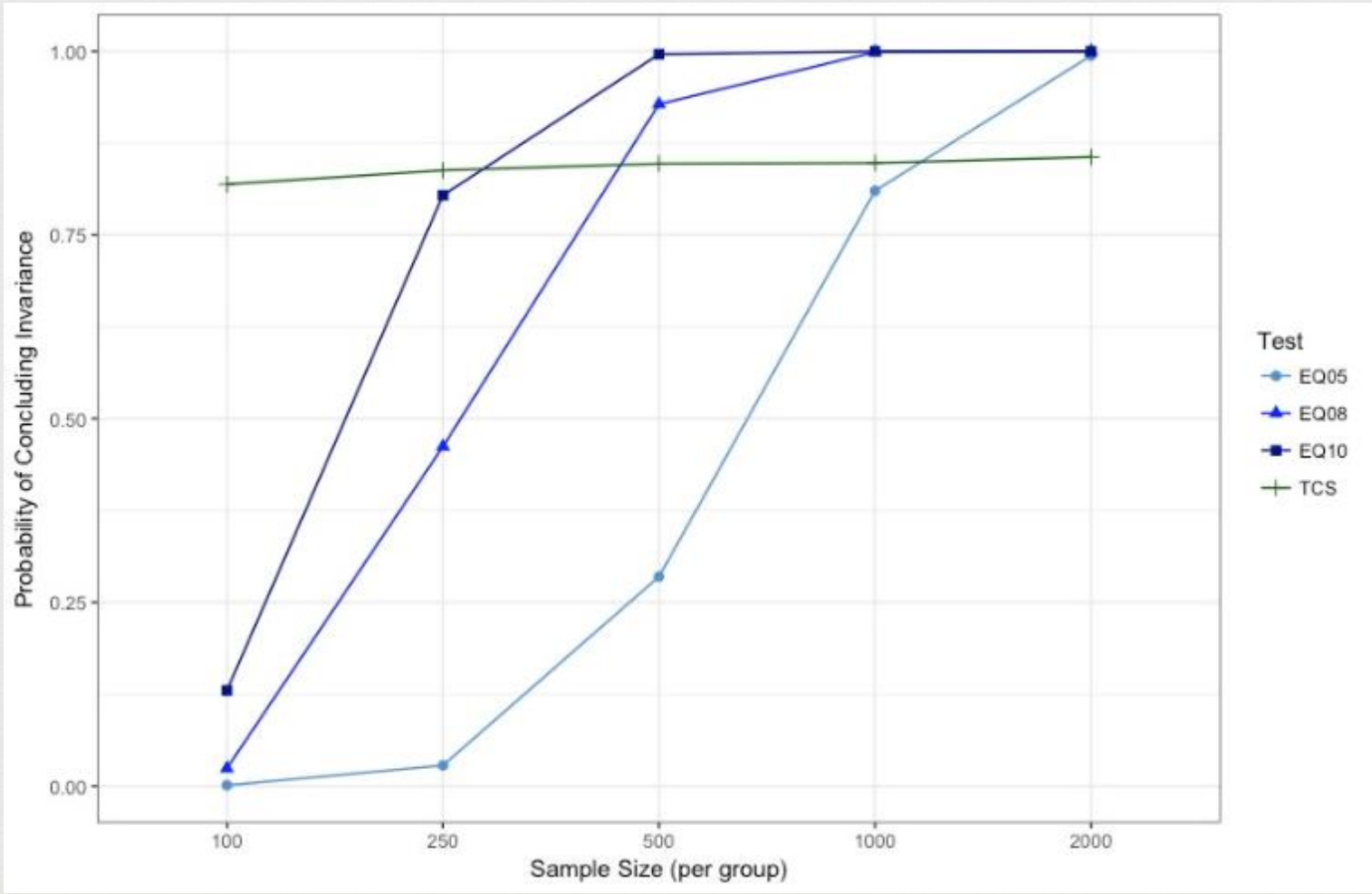
Power Results

Configural Invariance: 4 indicators, slightly different models



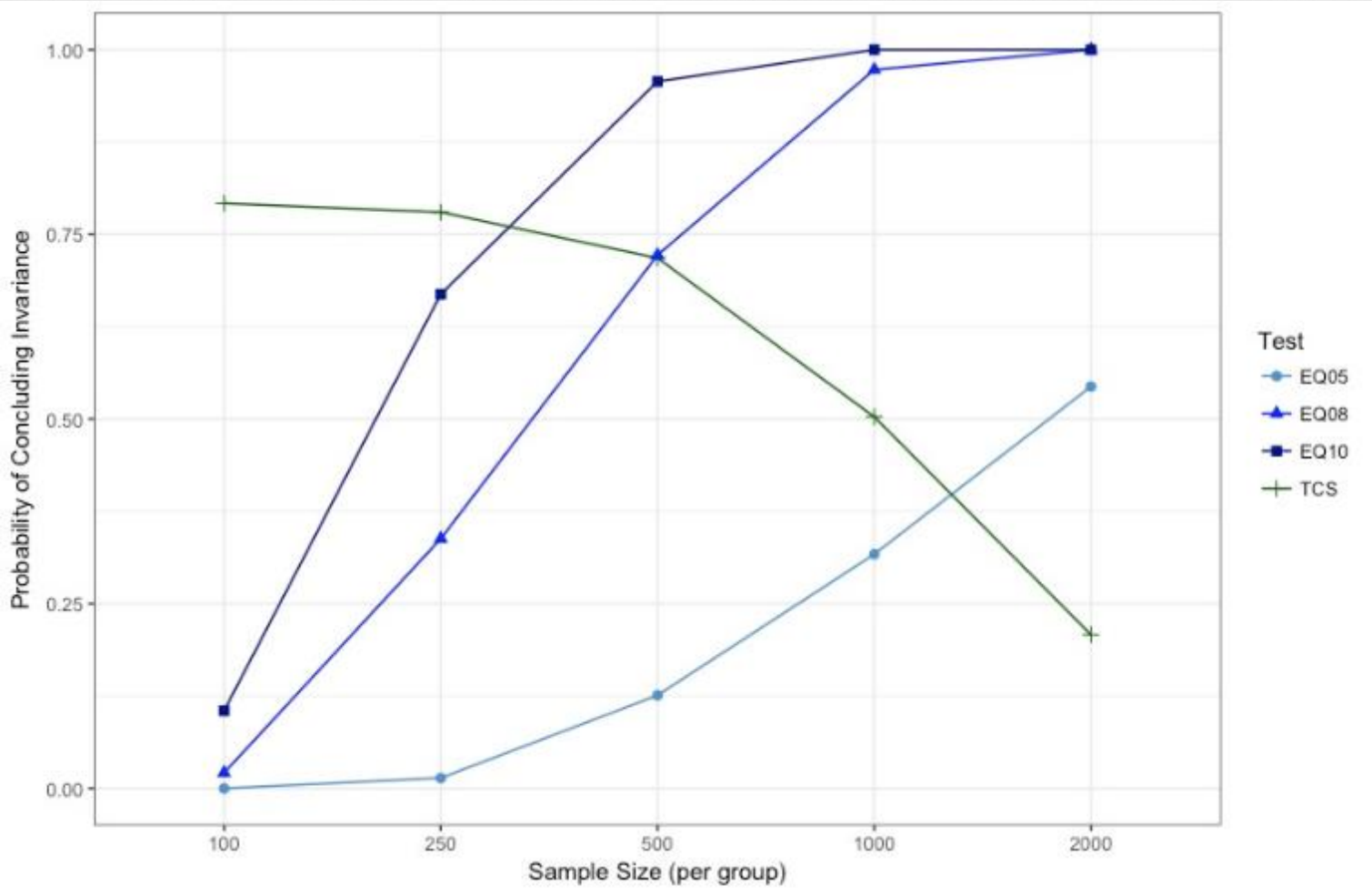
Power Results

Metric Invariance: 4 indicators, equal group models



Power Results

Metric Invariance: 4 indicators, slightly different models



Results Summary

- As expected, the χ^2 difference test results in illogical properties such as backwards power for finding invariance
- Yuan and Chan's EQ-A approach increased power at the expense of Type I error control
- The EQ method demonstrated good statistical properties, although power is low with very small sample sizes

- Power for finding invariance using fit indices depended on the degree of population misspecification
  - Performance of CFI cut-off strongly depended on condition

Conclusion

- The χ^2 difference test is not appropriate for establishing MI
- ΔGOF using a rigid adoption of common cut-offs is not recommended
- Equivalence testing is the logical statistical tool for testing MI
  - Yuan and Chan's (2016) adjusted RMSEA method is not recommended
    - No theoretical justification and very liberal Type I error rates
  - Some caution is required for the EQ approach with larger *ncp*s and small sample sizes

General Conclusion

- Researchers in Psychology frequently explore equivalence-based hypotheses
- Equivalence tests are rarely adopted because of unfamiliarity with the methods and lack of availability of software
- It is hoped that Psychology researchers will begin to recognize situations in which an equivalence test would be appropriate and that the software for conducting these tests becomes more available and user-friendly

Thanks!

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