Are you Teaching your Students Fairy Tales about Normality and Homoscedasticity? Introduce Robust Statistics Instead

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Discussion Overview

- Are Normality and Homoscedasticity Really Fairy Tales?
- Introduction to Robust Measures
- Do Robust Measures Make a Difference?
- Implementing Robust Statistics in the Classroom

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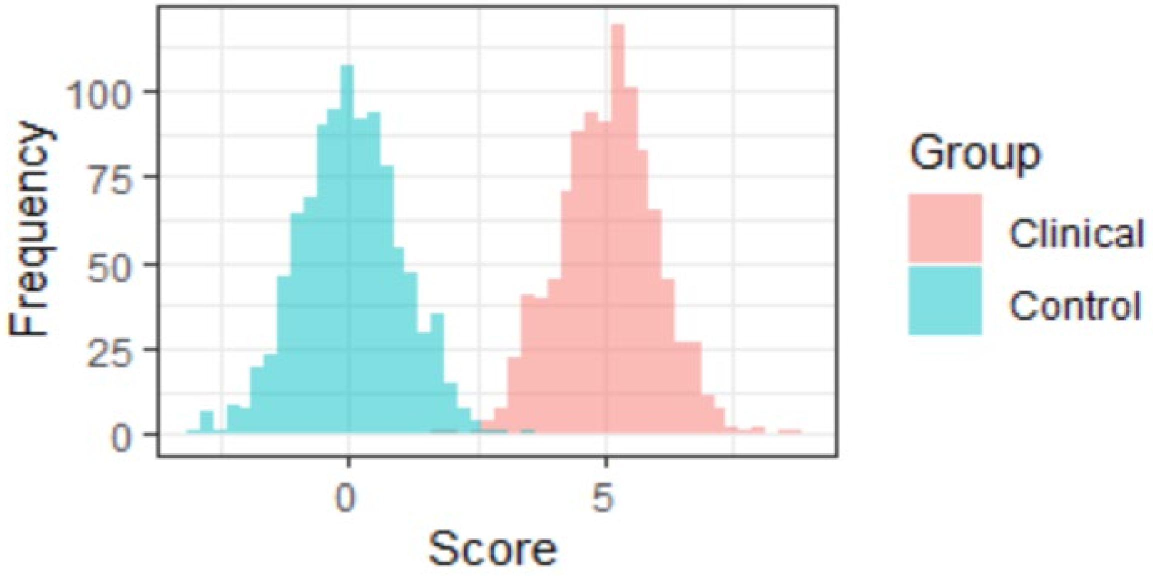
Are Normality and Homoscedasticity Really Fairy Tales?

Normality and Homoscedasticity



- Wouldn't it be great if all our distributions looked like this?
  - Normal distributions with equal variability
- The *Intro Stats Fallacy*

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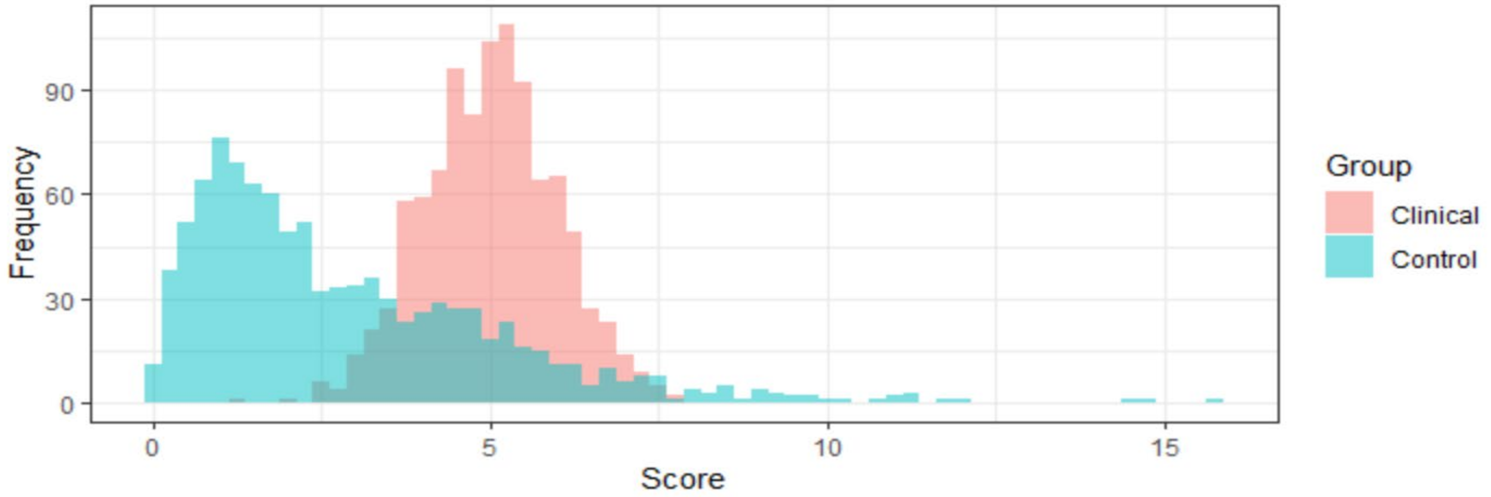
Normality and Homoscedasticity



- However, many studies have reviewed the distributions of variables in Psychology and found normality and homoscedasticity to be rare

The Unicorn, The Normal Curve, and Other Improbable Creatures

Theodore Micceri
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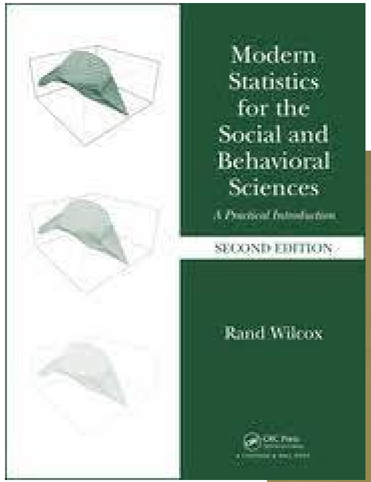
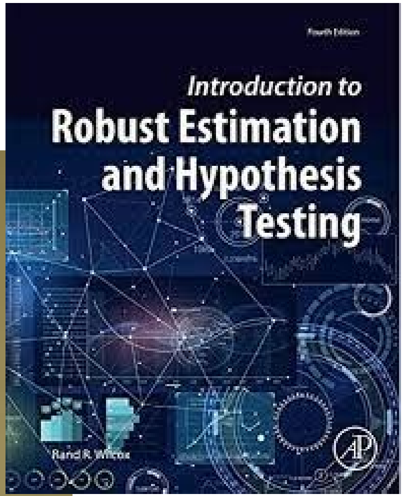
Introduction to Robust Measures

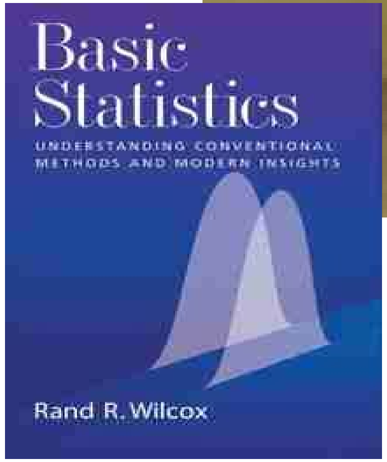
Rand Wilcox: Robust Statistics Guru!

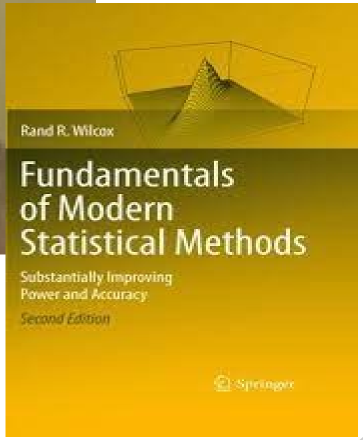
"Absolute Legend"

- Andy Field



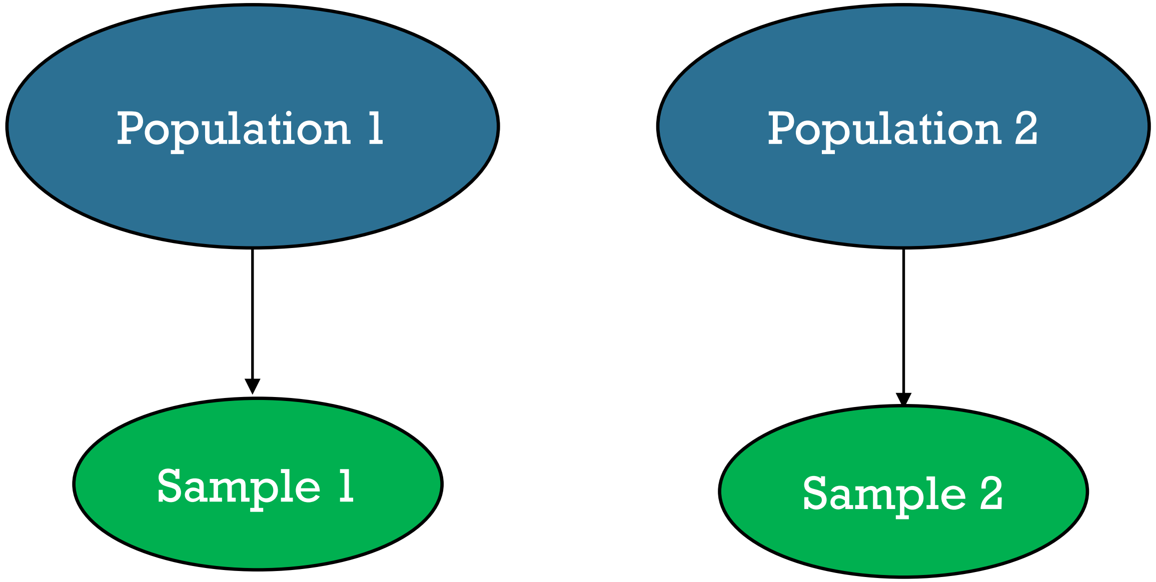






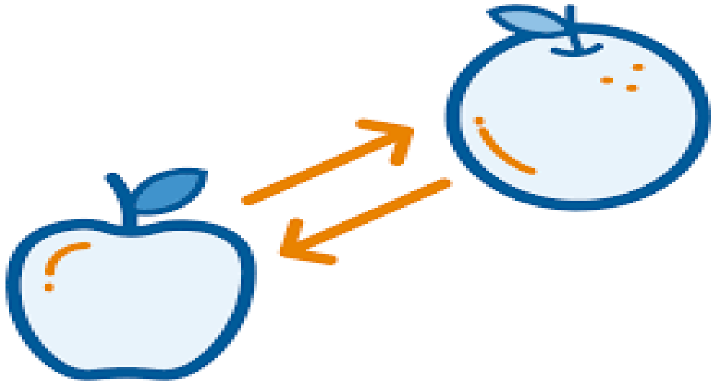
Brief Note: Design

I am going to focus exclusively on the **two independent samples design**



Brief Note: Transformations

Although **transformations** are an alternative to robust statistics, in addition to interpretation issues, they are not applicable in a wide range of situations



Introduction to Robust Statistics

- The primary goal of robust statistics is to ***limit the impact of superficial aspects of the data*** on the conclusions drawn from the data
  - For example, limit the effects of *nonnormality, outliers, unequal variances*, etc. on conclusions pertaining to the magnitude of effects
- This is often phrased in terms of analyzing the *bulk of the data*

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Central Tendency: Traditional Measure

- Mean (M)
  - Arithmetic average
  - $M = \frac{\sum X}{N}$

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Central Tendency: Robust Measures

- Median (Mdn)
  - Middle score of a sorted numeric variable
- Trimmed Mean (Mt)
  - Mean of a variable after removing extreme observations
    - 20% Trimmed Mean = mean after removing the most extreme 20% of observations from each tail

~~2 5 7~~ 12 14 15 15 15 17 21 22 25 34 ~~42 58 72~~

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Variability: Traditional Measure

- Standard Deviation (SD)
  - Square root of the average squared deviation from the mean
  - $SD = \sqrt{\frac{\sum(X - \bar{X})^2}{N-1}}$

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Variability: Robust Measures

- Median Absolute Deviation (MAD)
  - Median of the absolute deviations from the median
- Winsorized Standard Deviation (WSD)
  - SD after replacing the extreme observations with less extreme observations
    - 20% WSD = SD after replacing the lowest and highest 20% of cases from each tail with the lowest and highest nonextreme value

~~2 5 7~~ 12 14 15 15 15 17 21 22 25 34 ~~42 58 72~~

12 12 12 12 14 15 15 15 17 21 22 25 34 34 34 34

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Effect Sizes: Traditional Measure

- $$d = \frac{M_1 - M_2}{\sqrt{\frac{(n_1-1)SD_1^2 + (n_1-1)SD_1^2}{n_1+n_2-2}}}$$

Ordinary Cohen's *d*: Not Robust

Effect Sizes: Robust Measures

- $d_W = \frac{M_1 - M_2}{\sqrt{\frac{SD_1^2 + SD_2^2}{2}}}$

Welch Cohen's *d*: Robust to Variance Heterogeneity

- $d_r = .642 \frac{M_{t1} - M_{t2}}{\sqrt{\frac{WSD_1^2 + WSD_2^2}{2}}}$

Robust Cohen's *d*: Robust to Heteroscedasticity and Nonnormality

The multiplier .642 puts d_r on the same scale as a measure not incorporating WSD, with 20% trimming

Test Statistics: Traditional Measure

- $$t = \frac{M_1 - M_2}{\sqrt{\frac{(n_1-1)SD_1^2+(n_1-1)SD_1^2}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Ordinary *t* Test: Not Robust

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Test Statistics: Robust Measures

- $$t_W = \frac{M_1 - M_2}{\sqrt{\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}}}$$

Welch t Test: Robust to Heteroscedasticity

- $$t_Y = \frac{M_{t1} - M_{t2}}{\sqrt{\frac{(n_1 - 1)WSD_1^2}{h_1(h_1 - 1)} + \frac{(n_2 - 1)WSD_2^2}{h_2(h_2 - 1)}}}$$

h = sample size after trimming

Yuen t Test: Robust to Heteroscedasticity and Nonnormality

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Do Robust Measures Make a Difference?

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Descriptive Statistics

- Effects of Extreme Values on the Mean and Standard Deviation
  - X1 = {2 2 4 5 7 7 8 9 9 14}
  - X2 = {2 2 4 5 7 7 8 9 9 38}

	Mean	Mdn	M_t	SD	MAD	WSD
X1	6.7	7	6.4	3.7	3.0	4.7
X2	9.1	7	6.4	10.5	3.0	4.7

- The *breakdown point* of the M and SD is 1!

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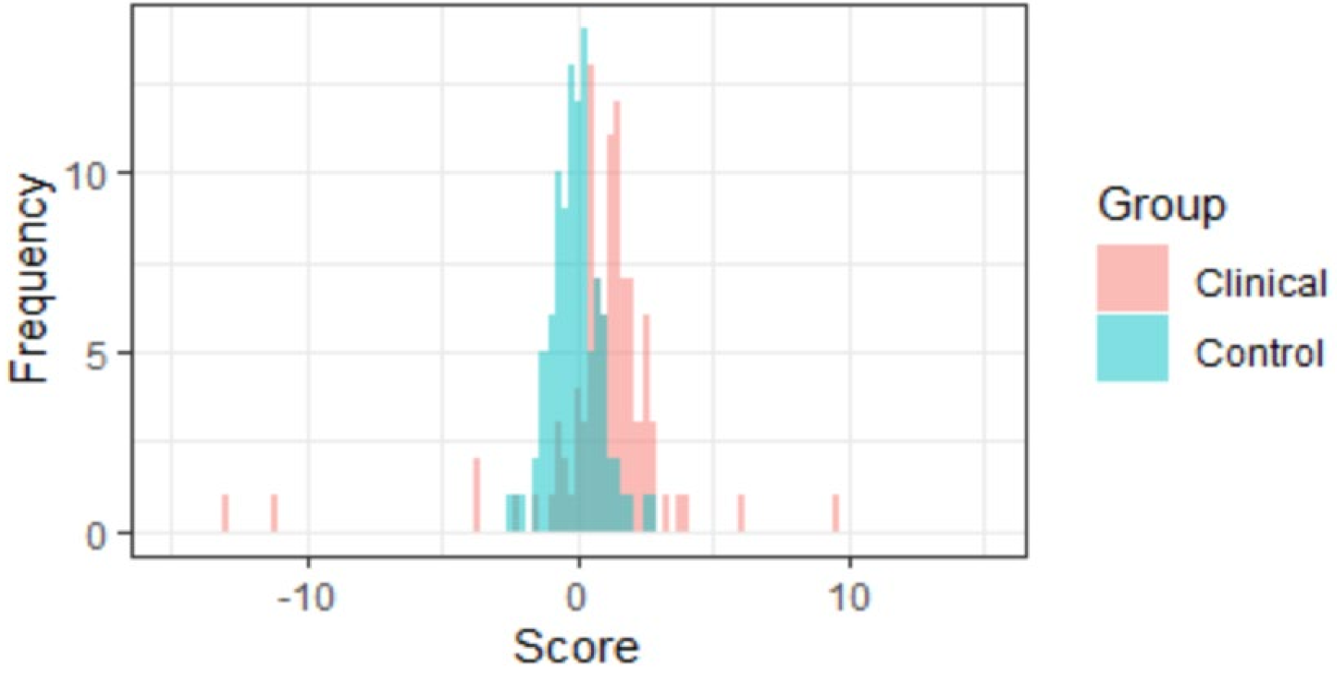
Effect Sizes

Effects of Nonnormality/ Heteroscedasticity on Cohen's *d*



- One group has some extreme scores
  - Imagine pre-post changes over an intervention for a control group and a clinical group

Effect Sizes

- **Effects of Extreme Scores on Cohen's *d***

- $$d = \frac{M_1 - M_2}{\sqrt{\frac{(n_1-1)SD_1^2+(n_1-1)SD_1^2}{n_1+n_2-2}}} = -.53$$

- $$d_r = .642\,\frac{M_{t1} - M_{t2}}{\sqrt{\frac{WSD_1^2+WSD_1^2}{2}}} = -1.40$$

Test Statistics

- Traditional *t* Test
  - Nonnormality and/or heteroscedasticity can have deleterious effects on the **Type I error rates** and **power** of traditional test statistics (e.g., *t*, *F*)
  - If NHST-based results are a primary outcome, robust test statistics are definitely recommended
    - E.g., Yuen test

Test Statistics

- *p*-Values for the Clinical vs Control Data
  - Ordinary *t* Test
    - $p = .051$
  - Welch *t* Test
    - $p = .009$
  - Yuen *t* Test
    - $p < .001$

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What can we do in the classroom?

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Undergraduate Introductory Statistics Class

- In an undergraduate *Introduction to Statistics* course, we might introduce them to:

Effect of nonnormality/outliers on measures of central tendency

Effect of nonnormality/outliers and heteroscedasticity on measures of effect size and NHST-based tests

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Undergraduate Introductory Statistics Class

- In an undergraduate *Introduction to Statistics* course, we might introduce them to:

Robust measures of central tendency and variability

Robust effect size measures

Graduate Statistics Courses

In a graduate *Statistics* course, we should be introducing students to the full complement of robust statistics

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What can we do in the Classroom?

- One way to allow students to visualize the advantages of robust statistics is via *Shiny* apps
  - Here is a tiny example demonstrating the effects of nonnormality and variance heterogeneity on traditional and robust effect sizes (Cohen's *d*)
- https://cribbie.shinyapps.io/Robust_Effect_Size/

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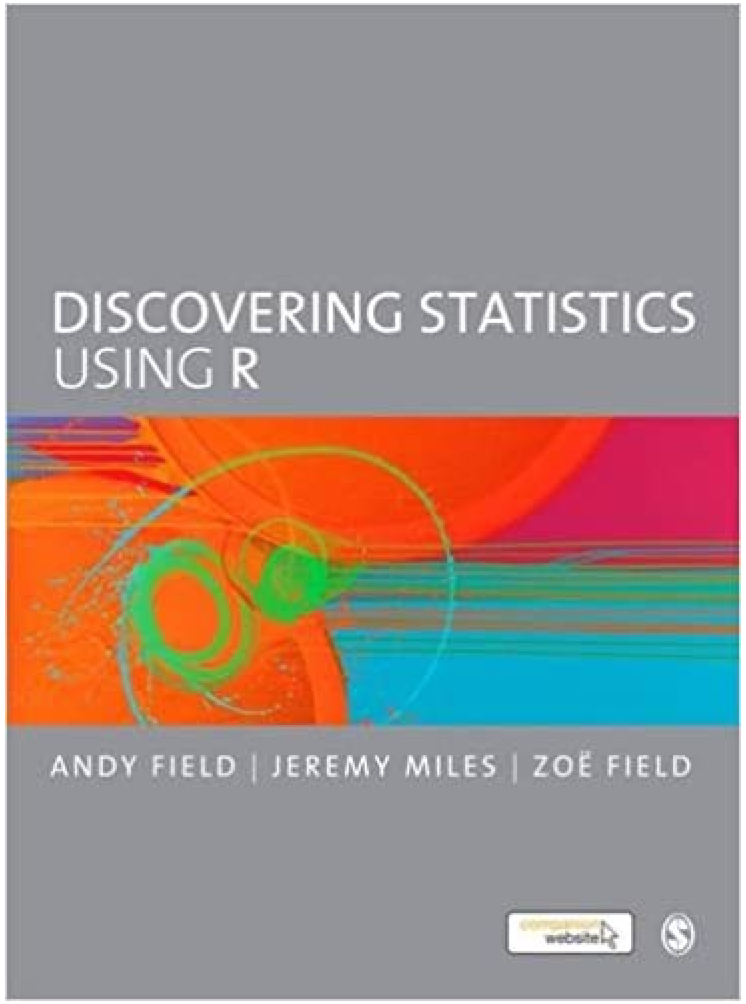
What if I use Statistical Software in my Course?

- Limitless robust statistics are available

- Trimmed Mean
  - mean(x, tr=.1)
- Median Absolution Deviation
  - mad(x)
- Winsorized Variance
  - WRS2::winvar(x, tr=.1)
- Robust Cohen's *d*
  - WRS2::akp.effect(model)
- Welch *t* Test
  - t.test(dv ~ iv)
- Yuen *t* Test
  - WRS2::yuen(dv ~ iv, tr=.1)

Textbook with Excellent Coverage of Robust Statistics!



9.5.2.7. Robust methods to compare independent means ②

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Thank You!