



Empirical Bayes

Introduction

- ▶ Empirical Bayes models estimate parameters using a prior probability distribution that is *estimated from the data*
 - ▶ In other words, the main difference between ordinary Bayes and empirical Bayes methods is that in standard Bayes the prior distribution (including the hyperparameters) are assumed to be known, whereas in the empirical Bayes approach the distribution/hyperparameters remain unknown and are estimated from the given data
- ▶ Thus, empirical Bayes is a *convenient* approach for establishing a prior (and setting hyperparameters)
- ▶ Empirical Bayes may be viewed as an *approximation* to a fully Bayesian model

Empirical Bayes vs Full Bayes

- ▶ Bayes Theorem

- ▶ $p(\theta|y) = \frac{p(\theta)p(y|\theta)}{p(y)}$

- ▶ In full Bayes, the prior distribution, including hyperparameters, are assumed known (or at least a guess of them is available)

- ▶ More specifically, the prior $[p(\theta)]$ depends on other parameters, η , the hyperparameters

- ▶ In empirical Bayes, the prior distribution, including the hyperparameters, are estimated from the data

- ▶ In essence, we are replacing $p(\theta|y)$ with $p(\theta|y, \hat{\eta}(y))$

Popularity of Empirical Bayes Methods

- ▶ Because empirical Bayes approaches estimate the prior from the data, many are skeptical of the approach
 - ▶ “There is nobody less Bayesian than an empirical Bayesian”
(Dennis Lindley)
- ▶ Empirical Bayes has not been as popular as fully Bayesian techniques recently, primarily due to advances in computational techniques for fully Bayesian analyses

When Can We Apply Empirical Bayes?

- ▶ To be able to apply empirical Bayes models, an appropriate dataset must be available
- ▶ More specifically, a dataset must be available that contains numerous 'replicates' that are similar in nature
 - ▶ E.g., insurance claims made in a year by clients
 - ▶ E.g., species of butterflies observed over a two-year period
 - ▶ E.g., counts of distinct words used by Shakespeare across many pieces of literature
 - ▶ E.g., removal of lymph nodes during cancer surgery

Empirical Bayes and Shrinkage

- ▶ Empirical Bayes results in the 'shrinkage' of parameters toward the average, with the usual property that the more data we have on the target the less the shrinkage will play a role
- ▶ Thus, empirical Bayes is an alternative form of shrinkage to:
 - ▶ Using priors in a full Bayesian analysis
 - ▶ Shrinkage in multilevel models

Empirical Bayes Example

- ▶ Estimating the win percentage of NHL goalies
 - ▶ Wins/Games Played (with a decision) = Wins/(Wins + Losses)
- ▶ Is the data appropriate?
 - ▶ The dataset has wins and losses for 876 NHL goalies
- ▶ Empirical Bayes approach
 - ▶ Use the data to estimate the hyperparameters (α , β) of a Beta distribution for the winning percentage
- ▶ Comparison
 - ▶ We can also compare this approach to a multilevel modelling and fully Bayesian approach